Chapter 4: Flow

In vivo, transportation of materials is controlled by the flow of a fluid such as liquid or gas. When the control of this flow does not work, life activity stagnates. In this chapter, we learn fundamental matters of flow dynamics, such as resistance of blood vessels. Technological analysis of the flow phenomena of the biological fluid *in vivo* is necessary for the design of the artificial heart and artificial blood vessel cooperating with a living body.

4.1 Fluid and solid

4.1.1 Fluid and pressure

In the stationary fluid, the force is uniformly transmitted to every direction. The vertical force at unit plane is called the "**pressure**". In each point of the stationary fluid, the force at unit plane is constant regardless of the direction of the plane. The "**stress**" in the solid (see 3.1.4), on the other hand, depends on the direction of the plane (**Fig. 4.1**).

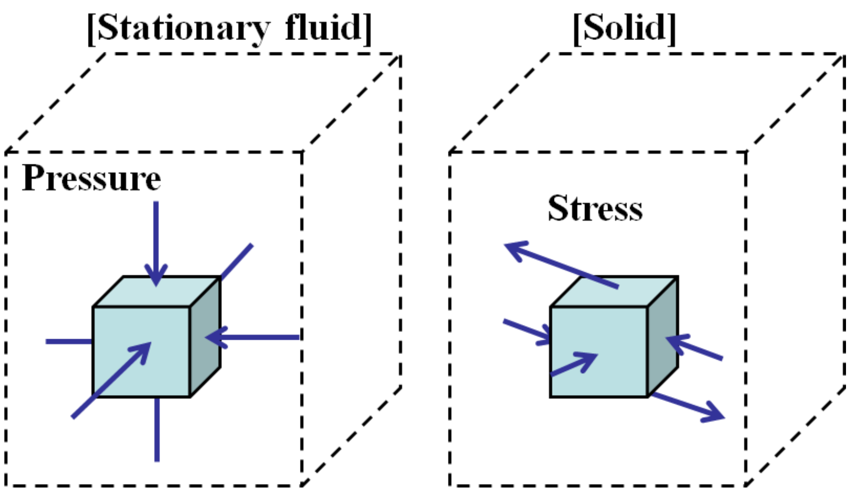

Fig. 4.1: Pressure and stress.

A connecting tube, which enables the movement of the liquid between two positions is called “**communicating tube**” (**Fig. 4.2**). The pressure transmission through the stationary liquid in the communicating tube is applied to the remote measurement of the pressure. The tube called a “**catheter**” is inserted into a blood vessel, filled with a saline solution (see 5.3.2), and applied to telemetry of the intravascular pressure.

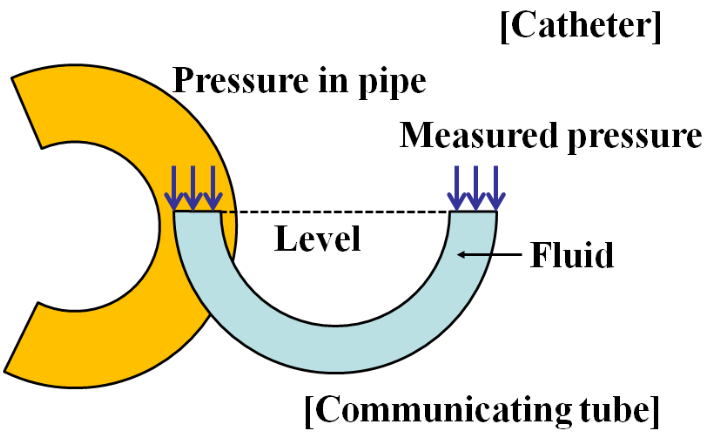

Fig. 4.2: Telemetry of pressure by the communicating tube.

The mass per unit volume is called “**density**”. The unit is kg m^{-3}. The fluid has uniform density in equilibrium. The volume of the “**compressible fluid**” is reduced by the pressure. The volume of the “**incompressible fluid**” is not reduced by the pressure.

The compressibility of the liquid is very small compared with that of the gas. The density of the incompressible fluid is constant regardless of the pressure.

When the density is constant, the **law of conservation of mass** leads to the law of conservation of volume. The law is described as the **continuity equation** 4.1 (**Fig. 4.3**).

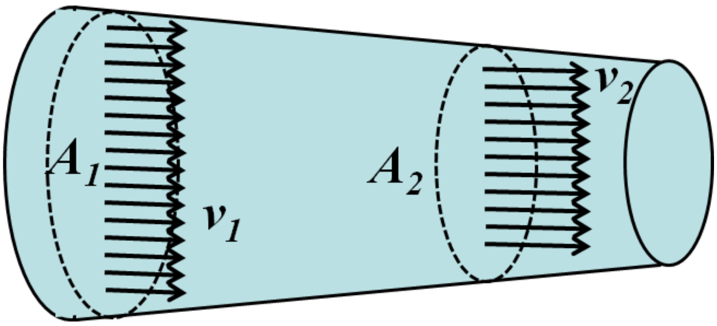

Fig. 4.3: Continuity.

$$A_1\, v_1 = A_2\, v_2 \qquad (4.1)$$

In the equation (4.1), A is the flow passage cross-sectional area [m^2], v is the flow velocity [$m\ s^{-1}$]. The equation shows that the flow velocity (v) increases, when the flow path cross-sectional area (A) decreases in the pipe.

The **equation of Bernoulli** (4.2) shows the law of conservation of mechanical energy in the form per unit volume.

$$(1/2)\,\rho\, v^2 + p = \mathrm{constant} \qquad (4.2)$$

In equation (4.2), ρ is the density [$kg\ m^{-3}$], v is the flow rate [$m\ s^{-1}$], and p is the pressure [Pa]. The pressure (p) decreases at the fast flow velocity (v), when ρ is constant in Eq. 4.2 (**Fig. 4.4**).

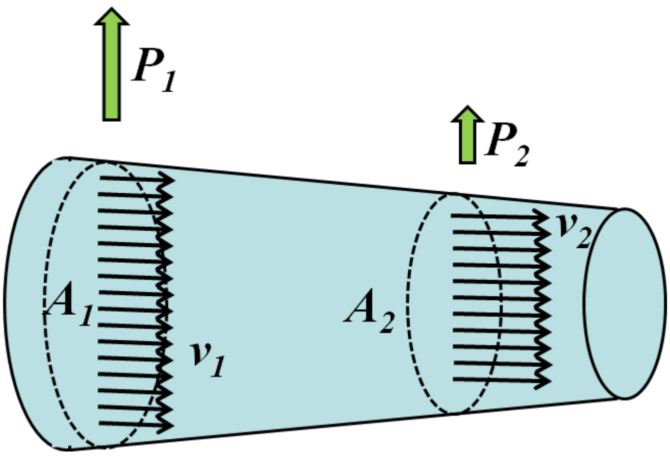

Fig. 4.4: Expression of Bernoulli.

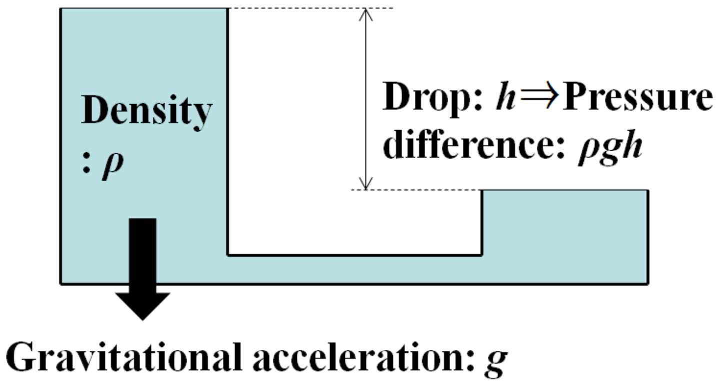

Fig. 4.5: Head drop.

In the gravitational field, fluid pressure p [Pa] is generated as the product of the density ρ [kg m^{-3}], the gravitational acceleration g [m s^{-2}] and the height h [m] (**Fig. 4.5**).

$$p = \rho\, g\, h \tag{4.3}$$

The pressure by the head is proportional to the density. The pressure by the head is different between the fluids of different density. The head control valve detects the fluid pressure difference between the fluids [3, 20]. The device can be applied to the flow control in the **shunt** (the connecting tube between the intracranial and the atrium or the peritoneal cavity) for hydrocephalus (**Fig. 4.6**).

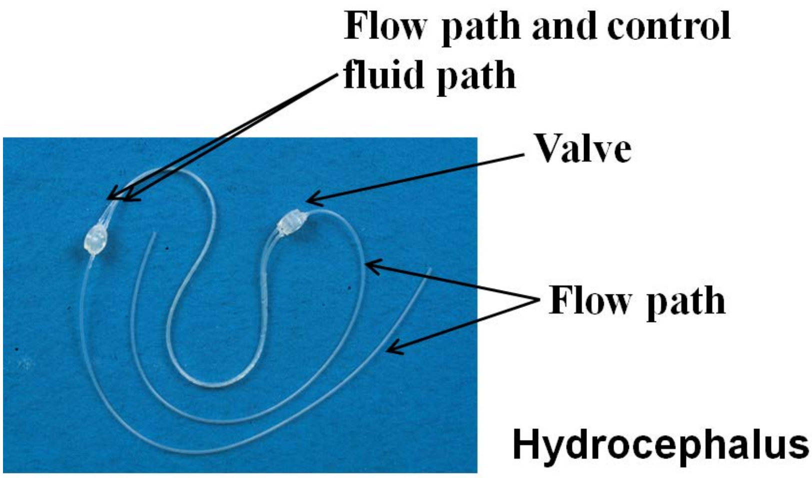

Fig. 4.6: Shunt.

The blood is drained according to the head with the **communicating tube**. The flow can pass over the higher position through the communicating tube by "**the principle of a siphon**" (**Fig. 4.7**).

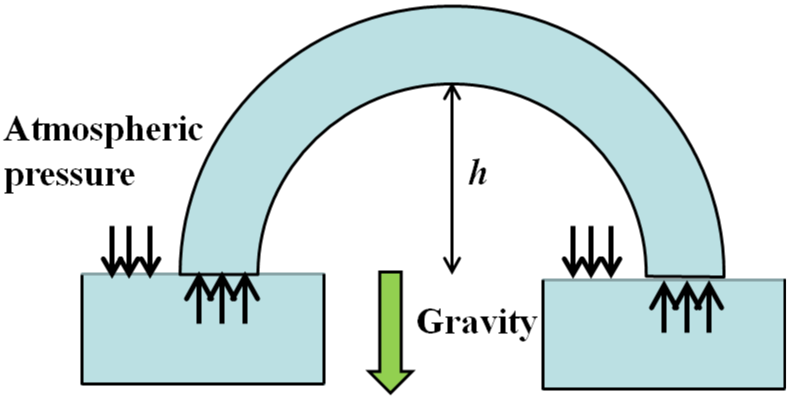

Fig. 4.7: Principle of siphon.

At the room temperature (298 K) and at 1 atm (101.325 kPa), consider the movement of water in accordance with the principles of a siphon (see the equation (4.3)). In the gravitational field, the water of 10 m height generates the following pressure:

$$1.0\times10^{3}\ \mathrm{kg\ m^{-3}}\times9.8\ \mathrm{m\ s^{-2}}\times10\ \mathrm{m}=98\ \mathrm{kPa} \qquad (4.4)$$

The water pressure at the point elevated by 10 m under 1 atm (101.325 kPa) is

$$101.325\ \mathrm{kPa}-98\ \mathrm{kPa}=3.325\ \mathrm{kPa} \qquad (4.5)$$

The vapor pressure of water is 3.140 kPa at 298 K.

At the point higher than 10 m at 298 K, the pressure of the water is below the vapor pressure, in which the water is vaporized. The water cannot be drained over the height of 10 m with the communicating tube.

The venous blood vessel wall has large compliance. When the amount of blood in the local vein decreases, the lumen becomes smaller. Successively, the walls of the vessel stick together, and the lumen is embolized. This is called “**collapse**” (**Fig. 4.8** (a)). The resistance of the flow path increases with the collapse.

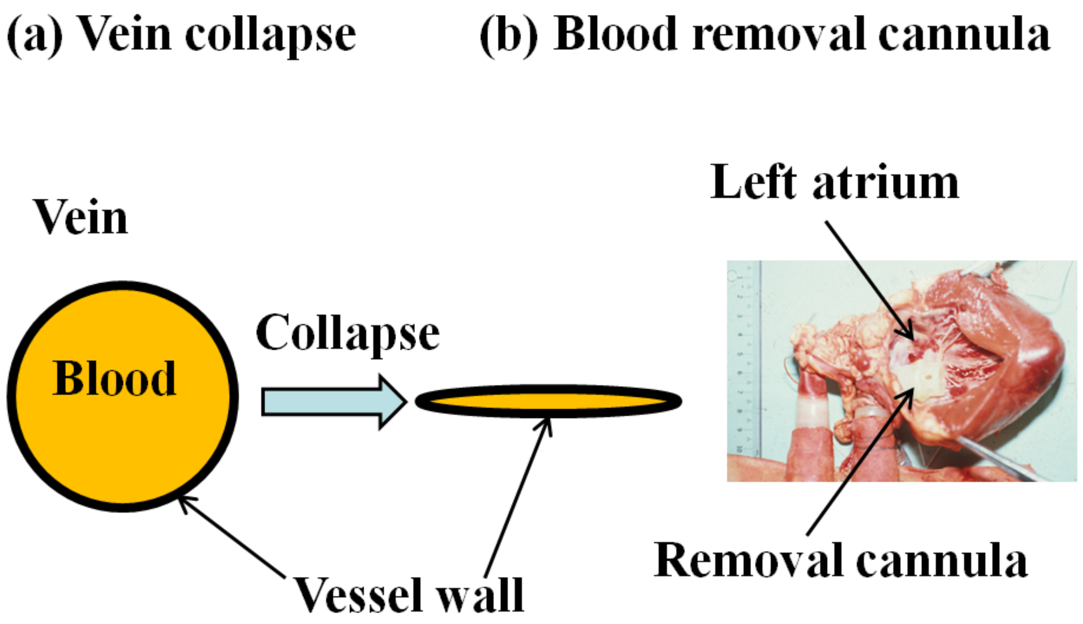

Fig. 4.8: Collapse.

The collapse should be avoided at the drainage of blood from the venous system. Several venous **cannula** tips are designed to avoid the collapse (Fig. 4.8 (b)) [21].

4.1.2 Elasticity and viscosity

When you hang a weight on a coil spring, the coil spring stretches. When you remove the weight on the coil spring, the coil spring returns to its original length. The “**Hook elastic body**” is the deformation model for the object similar to the coil spring (**Fig. 4.9**). The solid is approximated to the Hook elastic body. In the Hook elastic body, the "**strain**" is proportional to the “**stress**” (see 3.1.5).

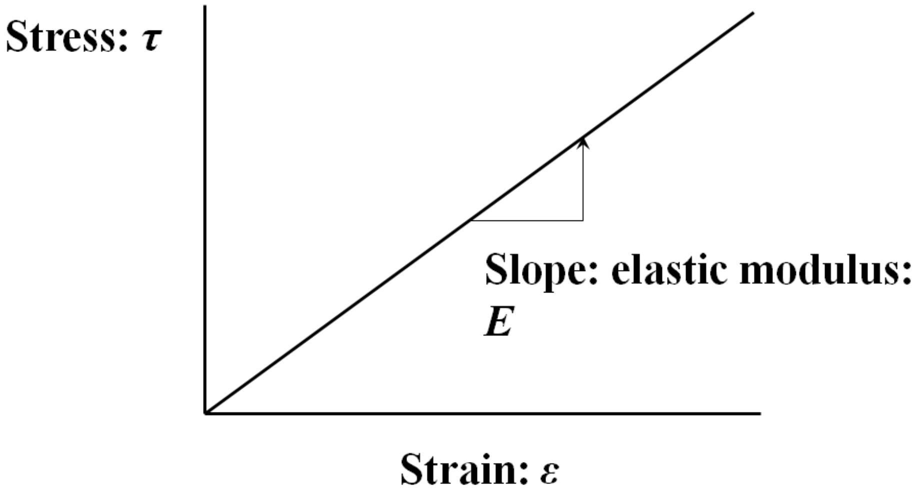

Fig. 4.9: Hook elastic body.

The water does not return to its original shape and position after removal of the force on the water. Although a small force is enough to make the slow flow, a large force is necessary to make the rapid flow.

Although the fluid such as liquid or gas is sheared by a force, it does not return to the original shape and position after removal of the force. In the fluid, the large force increases the "deformation rate".

The change of the strain per the unit of time is called the “**strain rate**”. The units of strain rate is s^{-1}.

The deformation rate is quantitatively expressed by the rate at the shear between the surfaces. The **shear rate** (γ) is the quotient (Eq. 4.6) of the speed difference between the surfaces (Δv) divided by the distance (y) between the surfaces (**Fig. 4.10**).

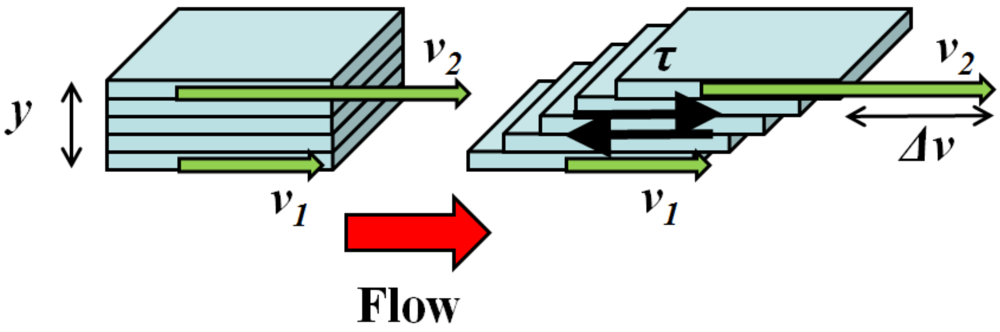

Fig. 4.10: Shear rate.

$$\gamma = \Delta v / y \tag{4.6}$$

Unit of the shear rate is s^{-1}. For example, the speed difference of 360 km per hour between two plates with the interval of 1 m makes the shear rate of 100 s^{-1}

The shear force per the unit area is called the **shear stress**. The unit of the shear stress is Pa. The quotient of the shear stress τ divided by the shear rate γ is called the **coefficient of the viscosity** η (Eq. 4.7).

$$\eta = \tau / \gamma \tag{4.7}$$

The unit of the coefficient of the viscosity is Pa s. The unit of poise [P] is sometimes used for the viscosity (Eq. 4.8).

$$1\ \mathrm{P} = 1\ \mathrm{dyn\ cm^{-2}\ s} = 0.1\ \mathrm{N\ m^{-2}\ s} = 0.1\ \mathrm{Pa\ s} \tag{4.8}$$

In the “**Newtonian fluid**”, the "shear stress" is proportional to the "shear rate" (**Fig. 4.11**). The Newtonian fluid is used as the simple model for the flow of the liquid or the gas.

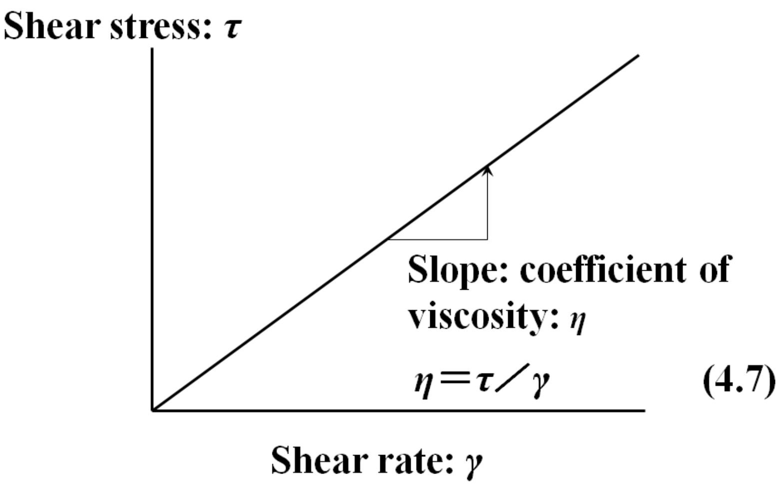

Fig. 4.11: Newtonian fluid.

The coefficient of the viscosity represents the flow resistance of the fluid, and depends on the temperature. In the liquid, the coefficient of the viscosity decreases with the increase of the temperature (**Fig. 4.12**). The coefficients of the viscosity of the water at 293 K and at 313 K are 1.002×10^{-3} Pa s and 0.653×10^{-3} Pa s, respectively. In the gas, conversely, the coefficient of the viscosity increases with the increase of the temperature. The coefficient of the viscosity of the gas is much smaller than the coefficient of the viscosity of the liquid. In the air, the coefficients of the viscosity at 298 K and at 323 K are 18.2×10^{-6} Pa s and 19.3×10^{-6} Pa s, respectively [22].

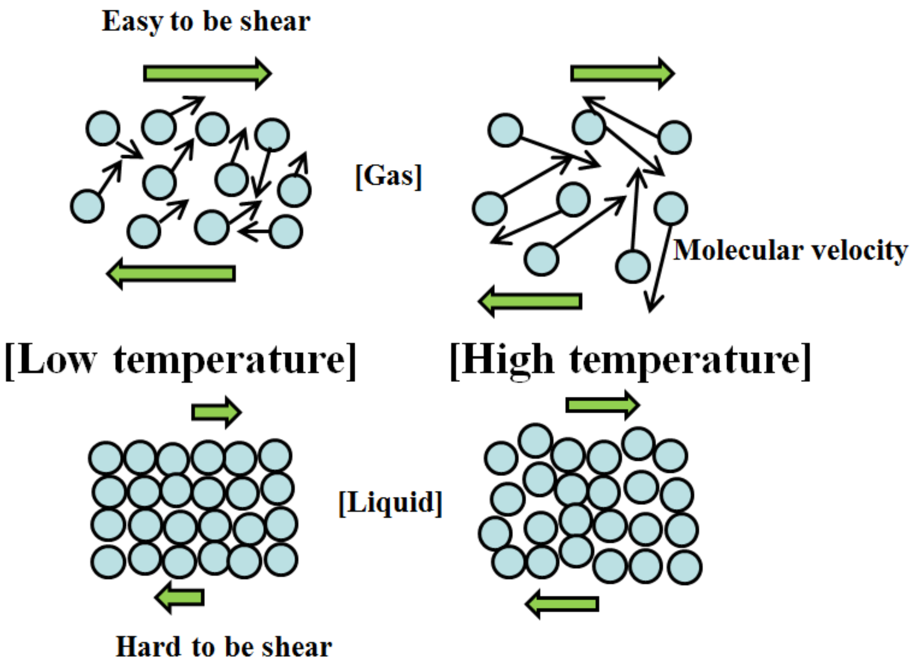

Fig. 4.12: Viscosity with temperature.

The viscosity can be explained by the interaction among the particles, which move as units in the fluid. The vibration of each particle at higher temperature induces shear flow between particles, which decreases the viscosity.

For example, the oil becomes smoother with the higher temperature, and thicker with the lower temperature. In the extracorporeal circulation, the viscosity of the blood increases at the lower temperature. The increase in viscosity is reduced by diluting the blood with the plasma substitute.

The coefficient of the viscosity of the gas, on the other hand, increases with the increase of the temperature. The velocity of the molecule of the gas increases with the increase of the temperature, which increases the interaction such as collision among the molecules. The interaction increases the coefficient of the viscosity. In the respiration, the lower temperature reduces the coefficient of the viscosity of the air, which reduces the flow resistance in the respiratory tract.

The blood has the higher coefficient of the viscosity at the lower shear rate. The shear stress is not proportional to the shear rate in the blood. The fluid, in which the coefficient of the viscosity varies with the shear rate, is called **non-Newtonian fluid** (**Fig. 4.13**).

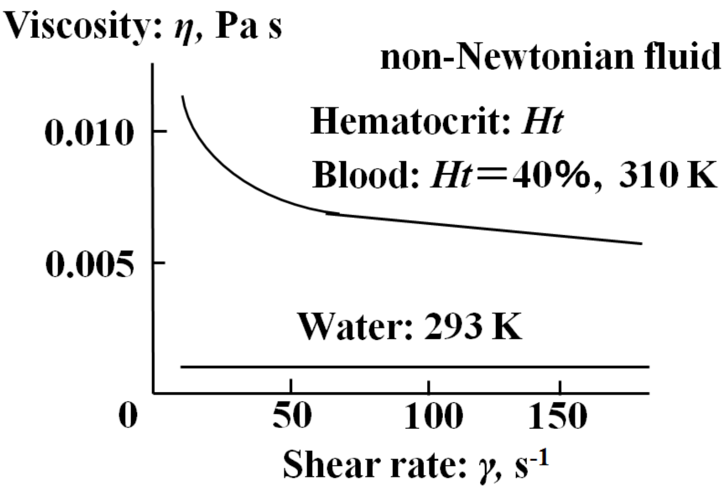

Fig. 4.13: Viscosity with shear rate.

The blood contains a huge number of red blood cells. The volume ratio of red blood cells is called **hematocrit** *Ht*. *Ht* is approximately equals to 40% in human. The value is a large number, which is the same level with the volume ratio of the hard spheres in the face-centered cubic lattice (see Question 3.3).

When the flow is slow, the interaction between the red blood cells increases the flow resistance. The interaction among the red blood cells decreases at the fast flow, which reduces the resistance of the flow. In the fast flow, the deformation of each erythrocyte is also contributes to the reduction of the flow resistance of the blood.

In the low shear region, red blood cells accumulate like a stack of coins, which is called **rouleau** (**Fig. 4.14**). The coefficient of the viscosity increases at the higher hematocrit. The viscosity of the blood changes with the thrombus formation. The change can be detected by a vibrating electrode (see 2.2.2), (**Fig. 4.15**) [23]. By the vibrating electrode, the differences can be detected not only on the impedance, but also on the viscosity of the yolk and the albumen (**Fig. 4.16**) [7].

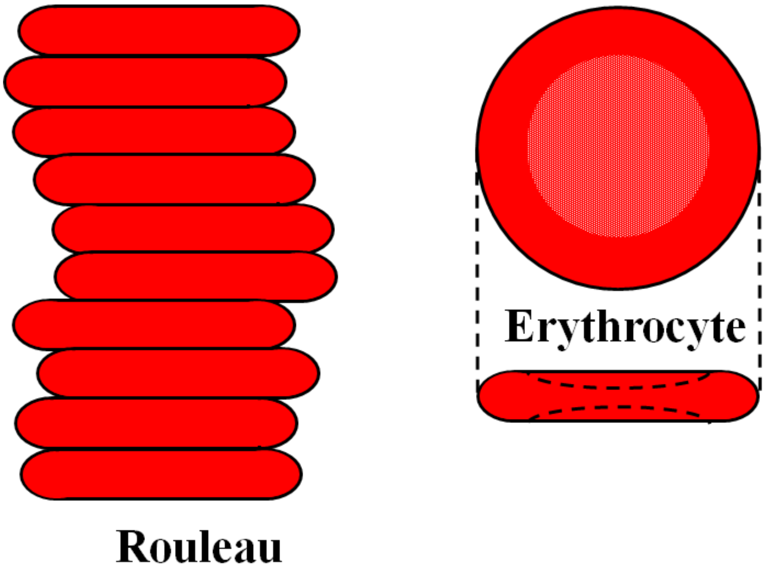

Fig. 4.14: Rouleau formation.

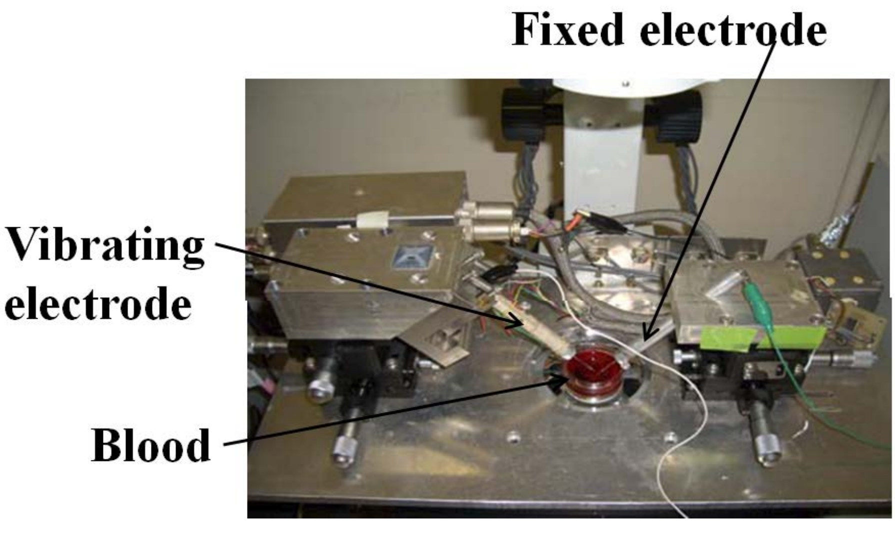

Fig. 4.15: Viscosity tracings with vibrating electrode.

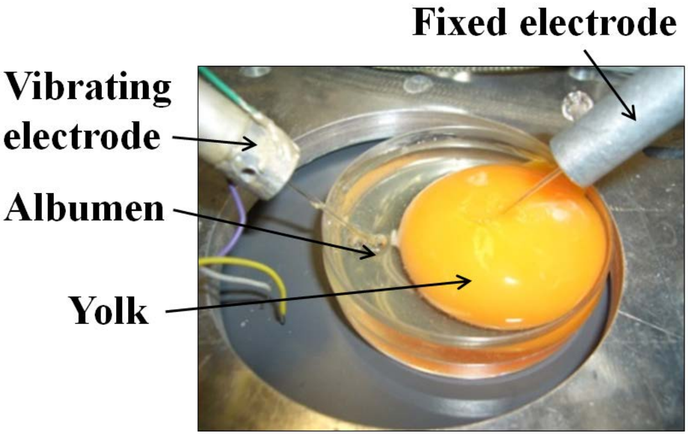

Fig. 4.16: Measurement of local viscosity with vibrating electrode.

4.1.3 Viscoelasticity

The most of objects cannot deform instantaneously, and can restore the force. They have the multi-properties of viscosity and elasticity. This multi-property is called viscoelasticity. The object, which shows viscoelastic property, is called viscoelastic body. The deformation behaviors of the polymeric materials and the biological tissues can be explained by **viscoelasticity**.

In **Maxwell model**, a viscous element is connected to an elastic element in series (**Fig. 4.17**). When a strain is applied to the model with a step function, a stress is generated in response to the strain at the elastic element. The stress governs the deformation speed at the viscous element. The deformation at the viscous element reduces the deformation at the elastic element. The reduction of the deformation at the elastic element decreases the stress with time. This phenomenon is called **stress relaxation**.

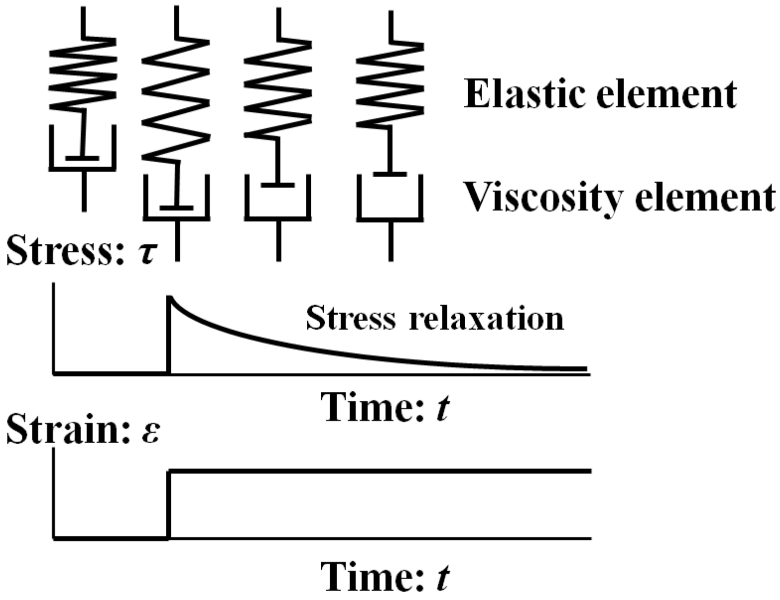

Fig. 4.17: Maxwell model.

In **Kelvin-Voigt model**, a viscous element is connected with an elastic element in parallel (**Fig. 4.18**). When a stress is applied with a step function, the deformation starts at the viscous element with a speed corresponding to the stress. The gradual increase of the strain at the elastic element increases the stress at the elastic element. The amount of increase of the stress at the elastic element reduces the amount of the stress at the viscous element. The reduction of the stress at the viscous element reduces the deformation speed at the viscous element with time. When all of the stress is supported by the elastic element, the deformation of the elastic element is saturated. The increase of deformation with time is called **creep deformation**.

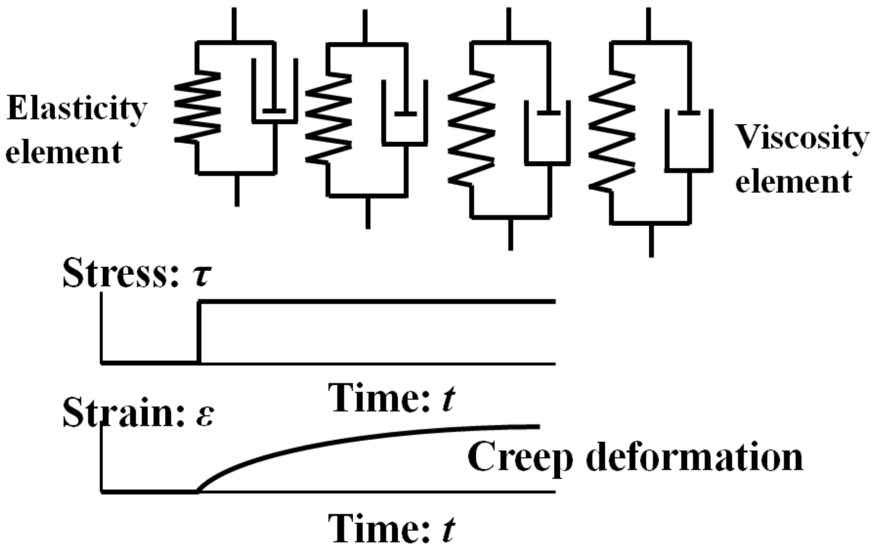

Fig. 4.18: Kelvin-Voigt model.

A polymer solution has elasticity as well as viscosity. Therefore, the solution spreads when it is released from the narrow flow path (Pallas effect). The surface of the solution near the rotating bar is lifted by the stress generated at the direction different from that of the shear stress (Weissenberg effect). The vortex and the turbulent flow resistance decrease (Toms effect) in the fluid.

4.2 Resistance of flow and distribution of velocity

4.2.1 Resistance of flow

Consider the resistance when the blood flows through a blood vessel. The flowing volume per unit time is called “**flow rate**”. The flow rate Q [m^3 s^{-1}] increases in proportion to the pressure difference ΔP [Pa] between the upstream and the downstream. In this case, the proportionality constant is the flow resistance Rf [Pa m^{-3} s].

$$Rf = \Delta P / Q \quad (4.9)$$

This relationship corresponds to "an electrical resistance R when an electric current flows through a conductive wire" calculated as a value of the "the potential difference ΔE between both ends of the conductive wire" divided by “the electric current I”.

$$R = \Delta E / I \tag{4.10}$$

“**Systemic circulation** resistance (total peripheral resistance) *Rs*” is a quotient of the difference between “aortic pressure *Pa*” and “right atrial pressure *Pr*” divided by "**cardiac output** *Qc*" (**Fig. 4.19**).

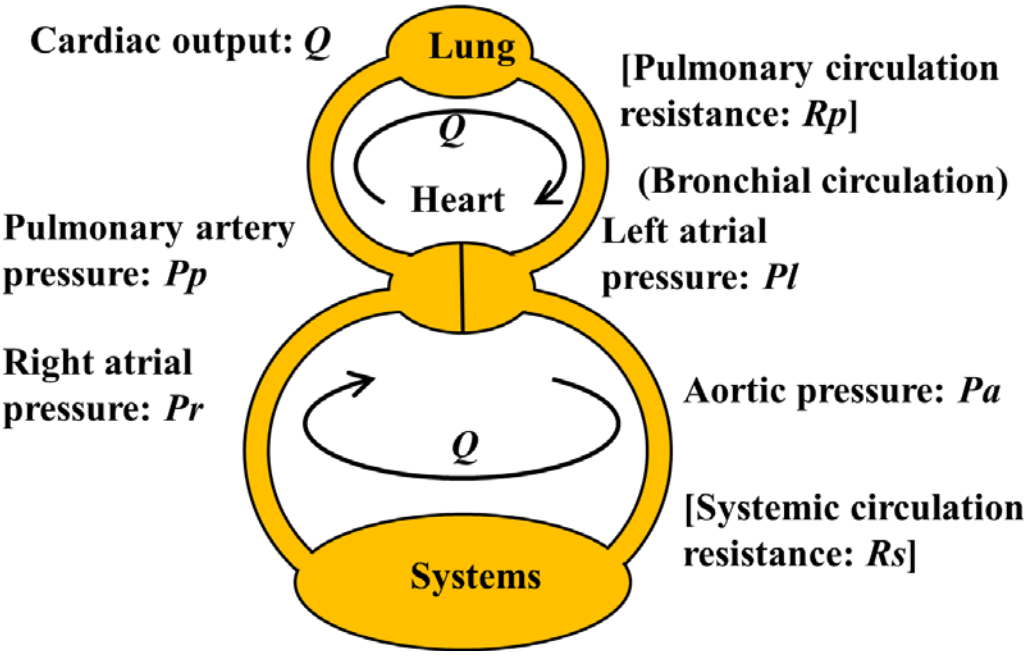

Fig. 4.19: Circulation resistance.

$$Rs = (Pa - Pr) / Qc \tag{4.11}$$

"**Pulmonary circulation** resistance (*Rp*)" is a quotient of the difference between "pulmonary arterial pressure (*Pp*)" and "left atrial pressure (*Pl*)" divided by "cardiac output (*Qc*)".

$$Rp = (Pp - Pl) / Qc \tag{4.12}$$

Since the pulmonary circulation has only one organ “the lung”, the resistance of the pulmonary circulation is lower than that of the systemic circulation: about one-fifth. Since "cardiac output" is the common at both left and right ventricles, the pressure difference at the pulmonary circulation is about one-fifth of that at the systemic circulation. When you count the **bronchial circulation** (from the aorta to the pulmonary vein), the output of the left ventricle is slightly bigger than that of the right ventricle.

Since the pressure difference [Pa] is divided by the flow rate [m^3 s^{-1}], the unit of flow resistance is [Pa m^{-3} s]. When [Pa] is replaced by the SI base unit [kg m^{-1} s^{-2}], the unit of flow resistance becomes [kg m^{-4} s^{-1}].

The aortic pressure of 13 kPa, the right atrial pressure of 1 kPa, and the cardiac output of 10^{-4} m^3 s^{-1} (= 6 l min^{-1}) make the systemic circulation resistance of 12×10^7 Pa m^{-3} s. With the similar equation, the pulmonary artery pressure of 3 kPa, the left atrial pressure of 1 kPa, and the cardiac output of 10^{-4} m^3 s^{-1} (= 6 l min^{-1}) make the pulmonary circulation resistance of 2×10^7 Pa m^{-3} s. In the pulsatile flow (see 4.3.1), both the pressure and the flow rate vary periodically according to the pulsation cycle.

During discharge, the high pressure ejects the fluid. During suction at the atmospheric pressure, on the other hand, 1 atm (101.325 kPa) is the maximum value for the pressure difference. For this reason, it is not easy to increase the flow rate during the suction. In the pumping of the heart, **diastole** is longer than **systole** (see question 4.6).

Electrical resistance R [Ω] of the metal wire is proportional to the length l [m], and is inversely proportional to the cross-sectional area A [m^2].

$$R = \rho l / A \tag{4.13}$$

The resistivity ρ [Ω m] is used to compare the magnitude of the resistance of metal.

The resistance of the flow increases with the length of the tube. The resistance of the flow decreases with the increase of the cross-sectional area of the tube. Is the resistance of the flow inversely proportional to the cross-sectional area (see 4.2.2)?

4.2.2 Hagen-Poiseuille flow

A flow, which has same velocity over the entire cross section of the circle pipe, is called "**Plug flow**". In the flow, the flow velocity in the center of the pipe is same as that of the vicinity of the pipe wall (**Fig. 4.20** (a)).

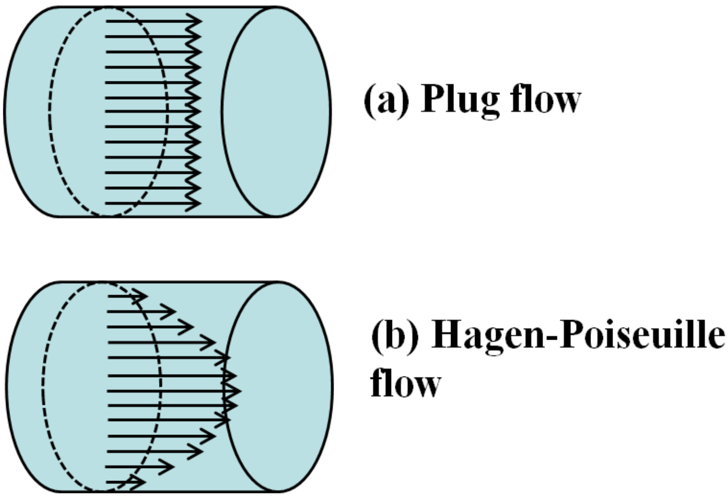

Fig. 4.20: Velocity distribution in pipe.

The flow velocity has the distribution at the cross section of the pipe, where the flow velocity of the central position is faster than that of the vicinity of the wall. A flow, which has the parabolic velocity distribution at the cross section of the circle pipe, is called "**Hagen-Poiseuille flow**". In Hagen-Poiseuille flow, the velocity vector is as follows: the velocity is the maximum at the center axis, and zero at the surface of the vessel wall. The velocity vectors are parallel and symmetric with respect to the axis of the pipe. The envelope of the velocity vectors makes parabola, where the tip corresponds to the maximum value (Fig. 4.20 (b)).

In the following equations, the flow velocity distribution in the Hagen-Poiseuille flow is introduced from a mechanical balance.

Let set concentric thin cylinders (length l, velocity v, radius r, and thickness dr) in the circular pipe (radius a, and length l) (**Fig. 4.21**). The speed of the inner cylinder is faster than that of the outer cylinder (Fig. 4.22). The difference of the speed makes the slip between cylinders. In the viscous flow, the slip makes the frictional force: with the same direction to the flow by the inner cylinder, and with the opposite direction to the flow by the outer cylinder, respectively. When the cylinder makes a uniform linear motion at a velocity v, the total force applied at the cylinder is zero. Therefore, "the force due to the pressure difference (ΔP) between the upstream side and the downstream side" balances with "the frictional force F between the cylinders". F is the product of the lateral area of the cylinder ($2\pi r l$) and the shear stress τ.

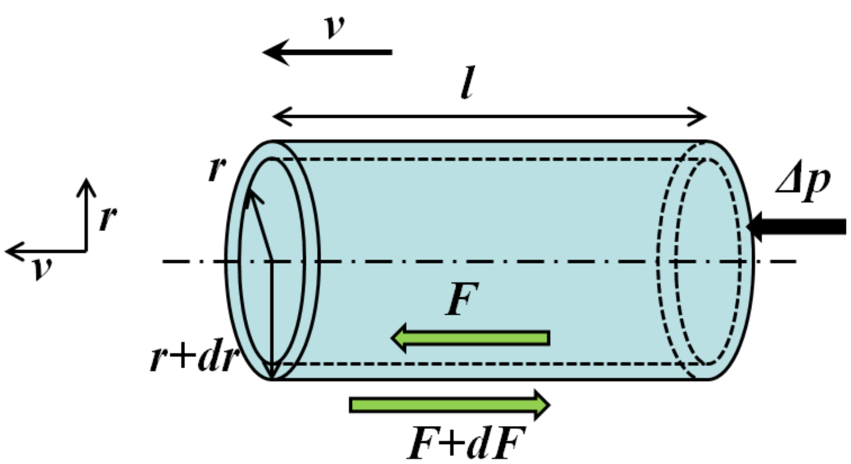

Fig. 4.21: Force balance in cylinder in flow.

$$F = 2\pi r l \tau \quad (4.14)$$

$$\tau = -\eta\, dv/dr \quad (4.15)$$

In Eq. 4.15, η is the coefficient of viscosity, and dv / dr is the shear rate. The origin of r is the center of the circular pipe. The velocity v has the maximum value at the center (r = 0). The velocity v decreases with the increase of r (**Fig. 4.22**). For this reason, Eq. 4.15 has a negative sign at the right side. Eq. 4.16 is introduced by substituting Eq. 4.15 into Eq. 4.14.

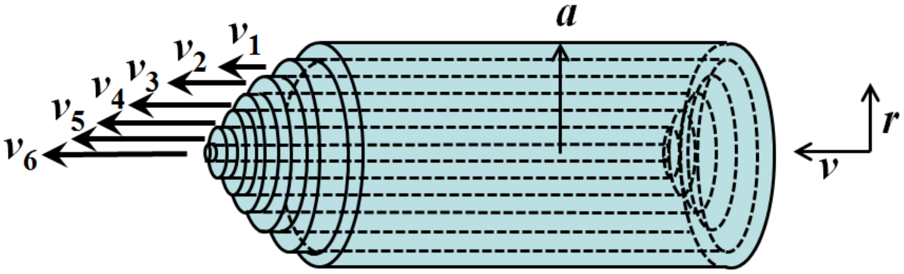

Fig. 4.22: Cylinders of fluid in flow through pipe.

$$F = -2\pi l \eta r \, dv / dr \qquad (4.16)$$

The frictional force dF, which is the difference between the frictional force with outer cylinder and that with the inner cylinder, resists the flow.

$$dF = -2\pi l \eta (1 / dr)(r \, dv / dr) \, dr \qquad (4.17)$$

The force dF balances with the force derived from the pressure difference ΔP between the upstream and the downstream, which are applied on the end surface area ($2\pi r \, dr$) of the thin cylinder.

$$dF = 2\pi r \, dr \, \Delta P \qquad (4.18)$$

Eq. (4.19) is derived from Eq. (4.17) and Eq. (4.18).

$$-2\pi l \eta(1/dr)(r\,dv/dr)\,dr = 2\pi r\,dr\,\Delta P \qquad (4.19)$$

Eq. (4.19) is rewritten to Eq. (4.20).

$$-(1/dr)(r\,dv/dr)\,dr = (\Delta P/(l\,\eta))r\,dr \qquad (4.20)$$

Both sides of Eq. (4.20) are integrated in r.

$$-\int(1/dr)(r\,dv/dr)\,dr = (\Delta P/(l\,\eta))\int r\,dr \qquad (4.21)$$

$$-r\,dv/dr = (\Delta P/(l\,\eta))(r^2/2) + C \qquad (4.22)$$

In Eq. (4.22), C is an integration constant.　C should be zero to apply the Eq. (4.22) at $r = 0$ (center of the pipe).　Thus, the Eq. (4.22) becomes Eq. (4.23).

$$r\,dv/dr = -(\Delta P/(l\,\eta))(r^2/2) \qquad (4.23)$$

The both sides of Eq. (4.23) are divided by r.

$$dv/dr = -(\Delta P/(l\,\eta))(r/2) \qquad (4.24)$$

The left side of Eq. (4.24) is the tangent inclination of the envelope of the velocity vectors.　dv/dr represents the shear rate γ.　γ is zero at the central axis, and the maximum at the wall of the pipe.

Shear rate at the wall (γw) is calculated at $r = a$, where a is the inner radius of the pipe.

$$\gamma w = -(\Delta P/(l\,\eta))(a/2) \qquad (4.25)$$

The both sides of the Eq. (4.24) are integrated by r.

$$\int dv = -(\Delta P/(l\,\eta))\int(r/2)dr \tag{4.26}$$

$$v = -(\Delta P/(4\,l\,\eta))r^2 + D \tag{4.27}$$

Equation (4.27) represents the flow velocity distribution in the cross section of the pipe. D is an integration constant. Since the flow velocity is zero at the wall, $v = 0$ at $r = a$.

$$D = (\Delta P/(4\,l\,\eta))a^2 \tag{4.28}$$

Eq. (4.27) combined with Eq. (4.28) makes Eq. (4.29).

$$v = (\Delta P/(4\,l\,\eta))(a^2 - r^2) \tag{4.29}$$

Eq. (4.29) represents a parabolic flow velocity distribution. The flow rate is calculated by integrating the flow velocity in the cross-section of the channel.

$$Q = \int_0^a 2\pi r\,dr\,\frac{\Delta P}{4l\eta}(a^2 - r^2) \tag{4.30}$$

$$Q = \frac{\pi\,\Delta P}{2l\eta}\int_0^a (a^2 r - r^3)dr \tag{4.31}$$

$$Q = \frac{\pi\,\Delta P}{2l\eta}\left[\frac{a^2 r^2}{2} - \frac{r^4}{4}\right]_0^a \tag{4.32}$$

$$Q = \pi a^4 \Delta P/(8\,l\,\eta) \tag{4.33}$$

Eq. (4.33) is rewritten to Eq. (4.34).

$$\Delta P = 8\,l\,\eta\,Q/(\pi a^4) \tag{4.34}$$

Eq. (4.34) is substituted into Eq. (4.25).

$$\Gamma w = -4\,Q / (\pi a^3) \tag{4.35}$$

Eq. (4.35) shows that the wall shear stress (Γw) is inversely proportional to the cube of the radius at the constant flow rate.

By substituting Eq. (4.34) into Eq. (4.9),

$$Rf = (8\,l\,\eta) / (\pi a^4) \tag{4.36}$$

In this distribution of flow velocities, resistance Rf [kg m^{-4} s^{-1}] of flow in the straight circular pipe is inversely proportional to the fourth power of the radius a [m]. When the radius is reduced by 16%, the resistance of the flow is doubled.

Can the flow rate is intuitively understandable with the Illustration of the flow velocity distribution? The inner radius of the circular pipe is doubled from (a) to (b) in **Fig. 4.23**. At the same flow velocity distribution, the four times bigger sectional area makes the four times bigger flow rate in "the plug flow" (Fig. 4.23 (a)). In "the Hagen-Poiseuille flow", on the other hand, the increased flow velocity at the center makes 16 times increase of the flow rate (Fig. 4.23 (b)). In Fig. 4.23 (b), the whole velocity distribution is extrapolated from the velocity distribution in the vicinity of the wall.

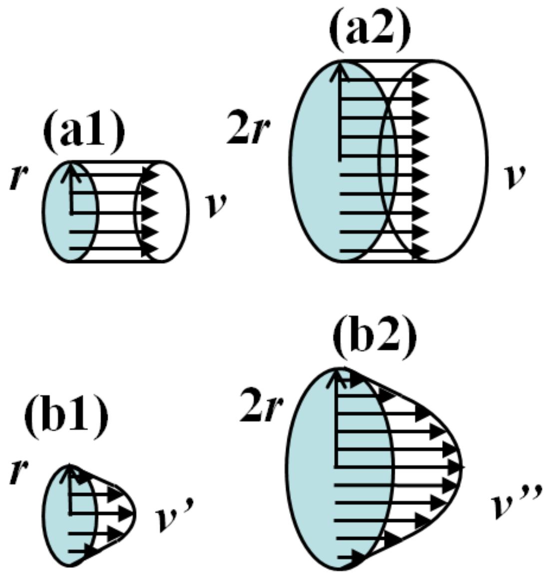

Fig. 4.23: Distribution of velocity.

When the flow velocity distribution of the Hagen-Poiseuille flow is applied to the human blood flow in the vessel with the inside diameter in each section (from 7×10^{-6} m to 0.025 m), the shear rate at the vascular wall is estimated by Eq. (4.35) as 60 ~ 800 s^{-1} [24].

The shear flow is one of the methods for continuous application of the mechanical stimulus to the cells. Vascular endothelial cells cover the inner surface of the vessel wall. Under stimulation of the shear flow at the wall, orientation has been observed so that the long axis of each cell makes orientation along the stream line. On the other hand, orientation of myoblasts has been observed perpendicular to the flow in the process of forming the myotubes by differentiation [16].

4.2.3 Requirement for Hagen-Poiseuille Flow (boundary conditions)

"Hagen-Poiseuille flow" can be applied to a steady laminar flow in a sufficiently long straight circular pipe without any branches (see 4.3.2). The velocity distribution in Fig. 4.20(b) is realized, when viscous forces between the layers are equilibrium in the entire flow in the circular pipe.

Vessels have bends and branches. The blood flow is pulsatile in the artery. Since blood is a non-Newtonian fluid, the viscosity varies with the shear rate. Thus, the flow velocity distribution deviates from that of the Hagen-Poiseuille flow. Even if above conditions were considered, the flow resistance drastically increases with the reduction of the inner diameter of the vessel.

The flow velocity distribution in the cross-section shifts from Fig. 4.20 (b) in the vicinity of the inlet of the pipe, because the flow velocity distribution of the upstream is taken over. At the downstream, the velocity distribution approaches to that of Fig. 4.20 (b). The section is called as "**inlet region**", and the length is called as "**inlet length**" (**Fig. 4.24**). The "inlet length" becomes shorter, when the radius *r* of the circular pipe or the Reynolds number *Re* (see 4.3.2) is small.

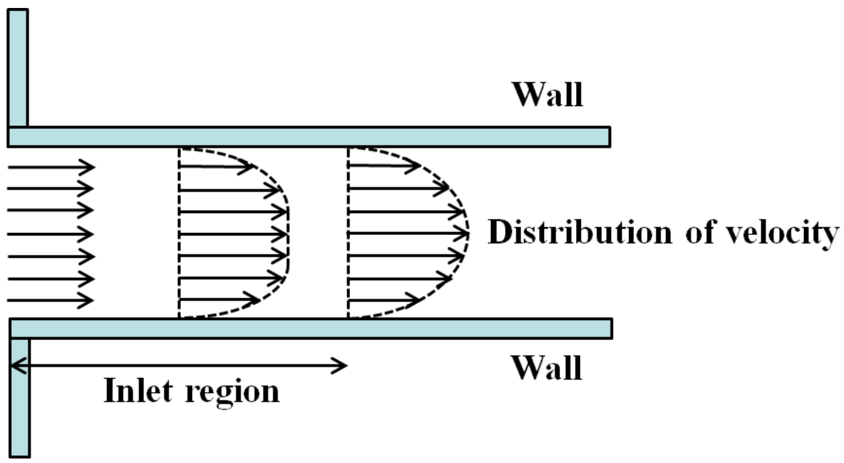

Fig. 4.24: Inlet region.

In the viscous flow, the flow velocity profile becomes parabolic in the vicinity of the wall surface. The area is called as the "**boundary layer**" (**Fig. 4.25**).

The viscosity coefficient of the fluid can be calculated from the flow resistance at the Hagen-Poiseuille flow through the capillary. Measurement of the viscosity by the capillary, however, is difficult on the blood of non-Newtonian fluid, because the flow velocity distribution deviates from the Hagen-Poiseuille flow. The viscosity of non-Newtonian fluid should be measured in the uniform shear field (see 4.2.4).

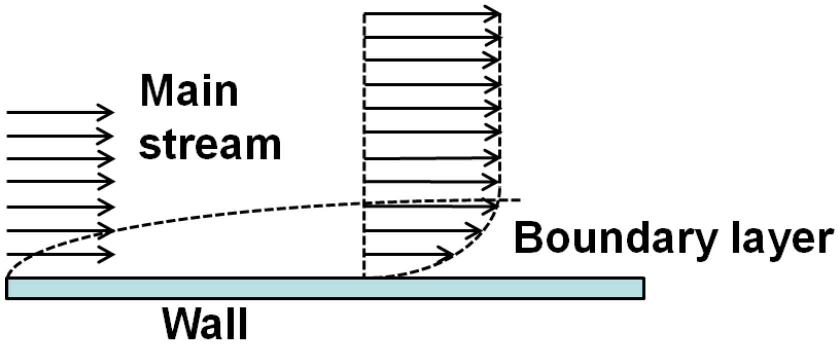

Fig. 4.25: Boundary layer.

4.2.4 Couette flow

Consider the flow of the fluid sandwiched between two walls with the distance of *d*. One of the walls is moving at speed of v, and the other wall is stationary. The fluid at the wall moves at the same velocity of that of the wall. The fluid at the stationary wall stops. The fluid sandwiched between the walls flows in the intermediate speed. When the velocity at each position increases proportionally to the distance from the stationary wall, the flow is called "**Couette flow**" (**Fig. 4.26**). In this case, the envelope of the velocity vector of the flow makes a straight line.

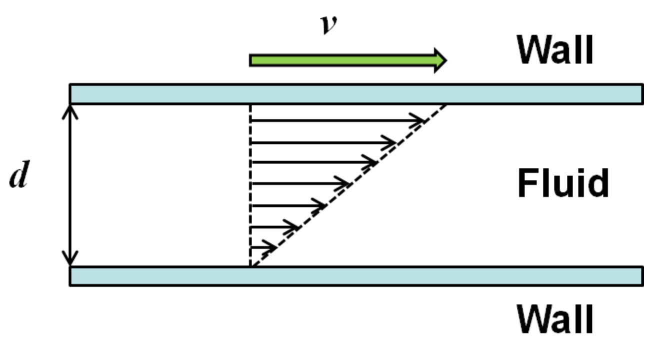

Fig. 4.26: Couette flow.

In the Couette flow, the shear rate γ is constant regardless of the distance from the wall.

$$\gamma = v / d \tag{4.37}$$

In the fluid, which is sandwiched between the rotating cone and the stationary plate, the flow velocity distribution of the Couette flow type occurs (**Fig. 4.27**). The distance d between the cone and plate increases in proportion to the distance r from the axis of rotation. When θ is very small,

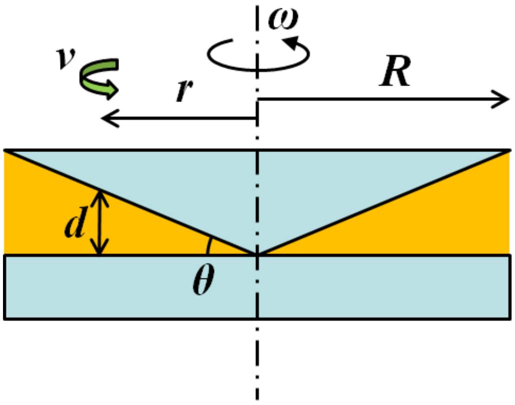

Fig. 4.27: Flow between rotating cone and stationary plate.

$$d = r \tan\theta = r\theta \tag{4.38}$$

θ is the angle [rad] between cone and plate. The speed v [m s^{-1}] of the conical surface is proportional to the distance r [m] from the axis of rotation.

$$v = r\,\omega \tag{4.39}$$

ω is the angular velocity [rad s^{-1}]. The shear rate γ is constant regardless of the distance r from the axis of rotation.

$$\gamma = v / d = r\,\omega / (r\,\theta) = \omega / \theta \qquad (4.40)$$

A uniform shear rate, which is suitable for the measurement of non-Newtonian fluids, is applied to the entire fluid (see 4.1.2). In the **cone-and-plate viscometer**, the fluid is sandwiched between the rotating cone and the stationary plate (**Fig. 4.28**). In the viscometer, moment (**torque**) to keep the rotation of the cone at a constant speed is proportional to the viscosity coefficient of the fluid (see Q. 4.9).

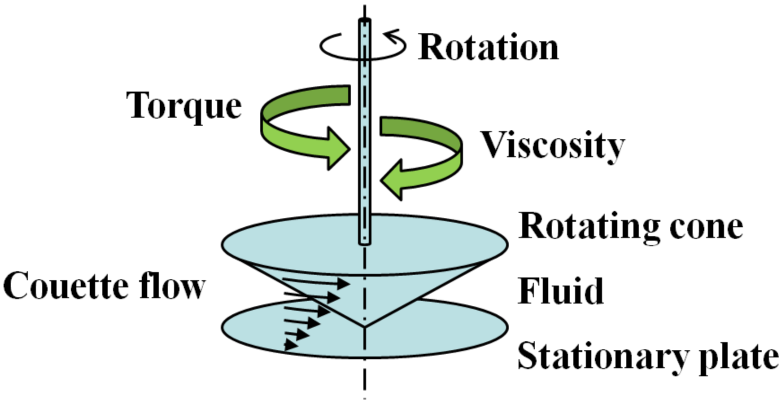

Fig. 4.28: Cone-plate viscometer.

The cone and plate viscometer can be applied to evaluate the effect of shear rate on clot formation, and to measure increase of the flow resistance with clot formation (**Fig. 4.29** (a)). The clotting time can be measured by the time t_0 before the rise of the torque. When the torque is increased from T_0 to T_1, the thrombus formation ability can be evaluated by the increase rate Rc (**thrombus ratio**) calculated by Eq. (4.41) (Fig. 4.29 (b)) [25].

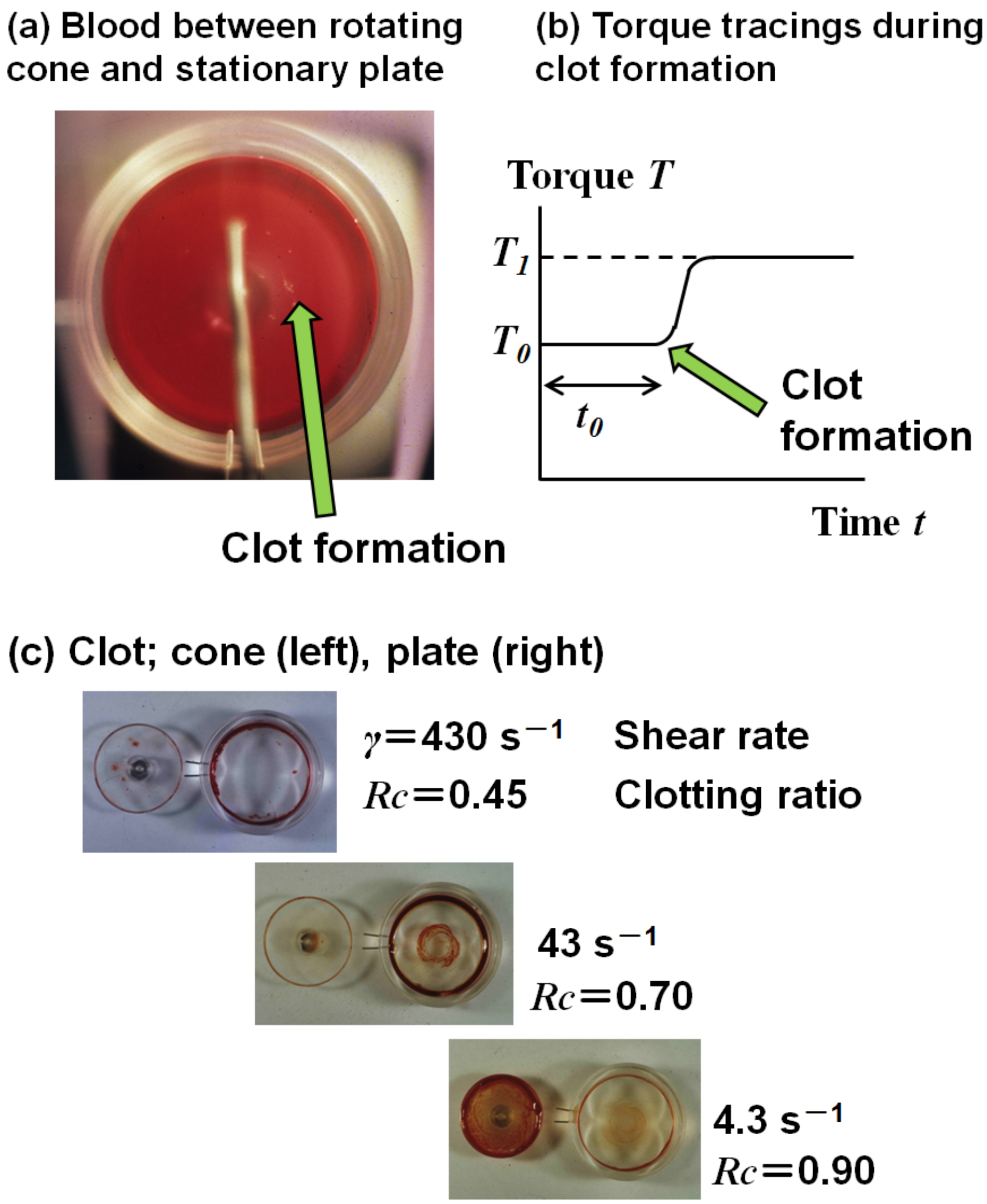

Fig. 4.29: Clotting between rotating cone and stationary plate.

(d) Shear rate γ and Clotting ratio Rc

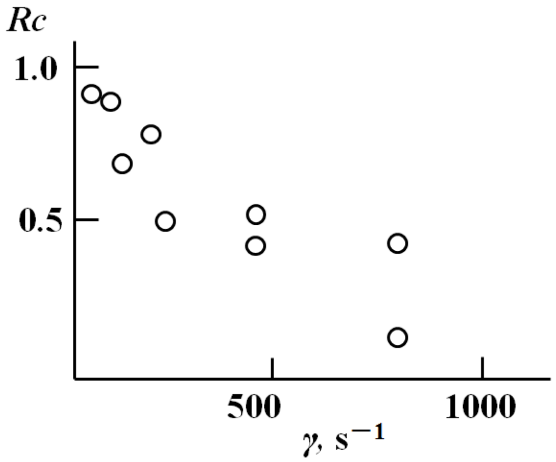

Fig. 4.29: Clot formation between rotating cone and stationary plate.

$$Rc = (T_1 - T_0) / T_1 \tag{4.41}$$

When the torque increases with clot growth, *Rc* approaches to unity (Fig. 4.29 (c)). When the shear rate is higher than 500 s^{-1}, *Rc* becomes lower than 0.5, which corresponds to the inhibition of the clot growth. When the shear rate is lower than 100 s^{-1}, on the other hand, *Rc* becomes higher than 0.7, which corresponds to the promotion of the clot growth (Fig. 4.29 (d)).

The stationary plane can be realized in the velocity distribution of the Couette type flow between the clockwise disk and the counterclockwise disk. The principle is applied to the "**counter rotating rheoscope**", in which the floating object in the Couette type flow is observed at the stationary plane (**Fig. 4.30** (a, b)). The deformation of the floating object can be observed in the shear field of the fluid in the device [4].

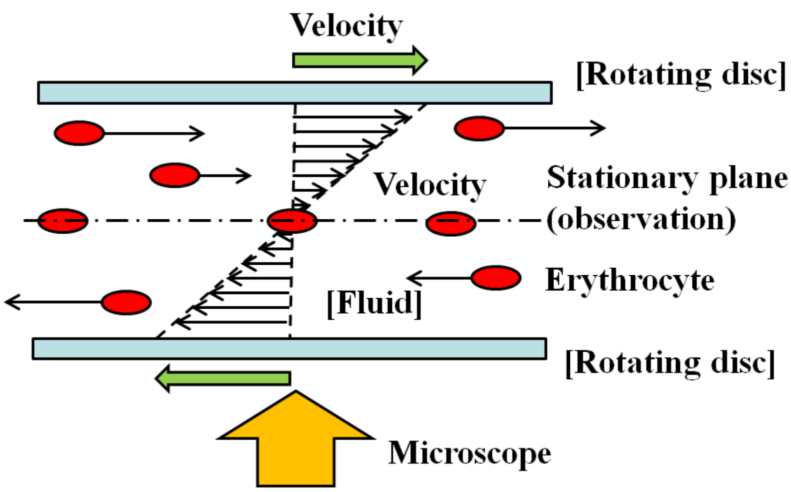

Fig. 4.30(a): Counter rotating rheoscope.

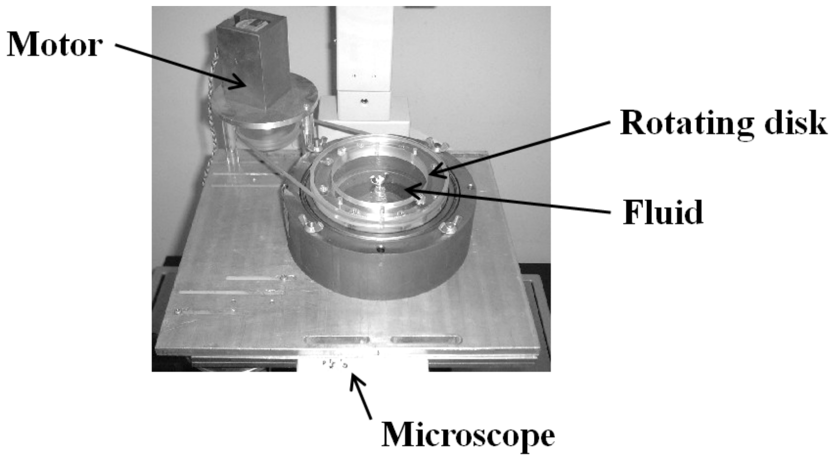

Fig. 4.30(b): Counter rotating rheoscope.

The deformation of erythrocytes can be observed during suspension in the shear field at “counter rotating rheoscope”. The deformability of erythrocytes changes from generation in the bone marrow, while they circulate through the blood vessels. Deformability of erythrocytes varies with the contents. Deformability of each erythrocyte can be measured, after sorting according to their density by centrifugation (see 6.1.1) [26].

4.2.5 Flow between parallel walls

The flow velocity distribution occurs between parallel plate walls: the flow velocity is small near the wall, and maximum at the center. The wall shear rate γ in the flow velocity distribution, as shown in **Fig. 4.31** (a), is able to be calculated by the similar equations as in 4.2.2. In Fig. 4.31 (a), the parallel velocity vectors are symmetrical relative to the center plane between the parallel walls (with distance d). In the flow direction plane vertical to the wall, the envelope of the velocity vectors makes the parabola: the velocity is the maximum at the center of apex, and zero at the wall.

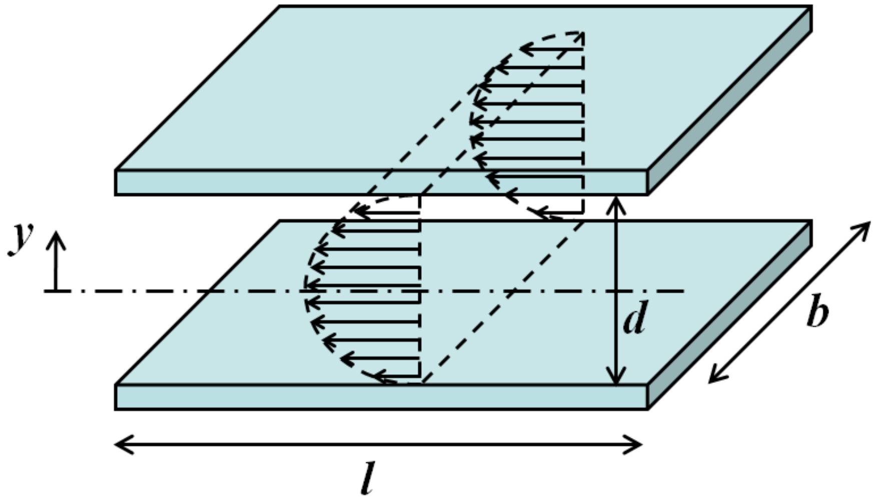

Fig. 4.31(a): Velocity distribution in flow between parallel walls.

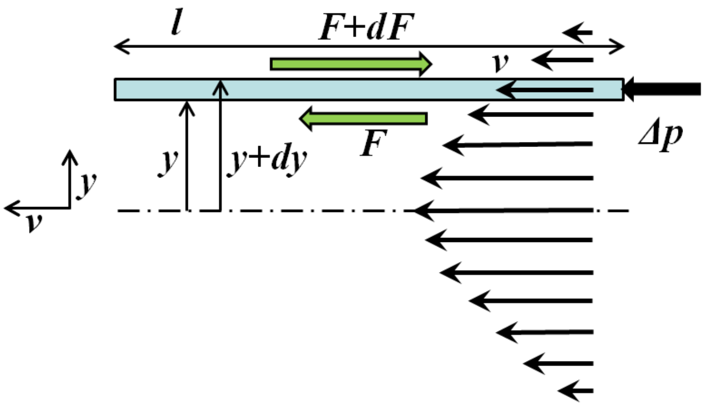

Fig. 4.31(b): Force balance in flow between parallel walls.

The origin is defined at the center between the parallel walls. The y-axis is defined vertically towards the wall. A virtual thin flat plate sandwiched between y and $y + dy$ has the velocity of v, the width of b, and the length of l (Fig. 4.31 (b)). When the plate has uniform linear motion, "the friction force ΔF between the plates" balances with "the force by the pressure difference ΔP between the upstream and the downstream". The friction force F is the product of the friction area of the plate ($b\ l$) and the shear stress τ.

$$F = b\ l\ \tau \tag{4.42}$$

$$\tau = -\eta\ dv / dy \tag{4.43}$$

In Eq. (4. 43), η is the viscosity coefficient of the fluid. At the center ($y = 0$), v is maximum. The right side of the Eq. (4.43) has the negative sign, because the velocity v decreases with y.

By substituting the Eq. (4.43) into Eq. (4.42),

$$F = -b\ l\ \eta\ dv / dy \tag{4.44}$$

The difference of the frictional force dF between the outside of the thin flat plate and the inside of the thin flat plate is the force resisting the flow.

$$dF = -b\ l\ \eta (1 / dy)(dv / dy) dy \tag{4.45}$$

dF balances with the force by the pressure difference ΔP between the upstream and the downstream loading at the end face of the thin flat plate (area of $b\ dy$).

$$dF = b\ dy\ \Delta P \tag{4.46}$$

Combining Eq. (4.45) and Eq. (4.46),

$$-b\,l\,\eta(1/dy)(dv/dy)dy = b\,dy\,\Delta P \quad (4.47)$$
$$-(1/dy)(dv/dy)dy = (\Delta P/(l\,\eta))dy \quad (4.48)$$

Both sides of Eq. (4.48) are integrated by y,

$$-\int(1/dy)(dv/dy)dy = (\Delta P/(l\,\eta))\int dy \quad (4.49)$$
$$-dv/dy = (\Delta P/(l\,\eta))y + C \quad (4.50)$$

In Eq. (4.50), C is an integration constant. $C = 0$, because $dv/dy = 0$ at $y = 0$ (at the center between the walls). Therefore, Eq. (4.50) becomes Eq. (4.51).

$$dv/dy = -(\Delta P/(l\,\eta))y \quad (4.51)$$

The left side of Eq. (4.51) is the tangent slope of the envelope of the velocity vector, and represents the shear rate γ ($=dv/dy$). γ is zero at the center, and the maximum at the wall.

The shear rate at the wall (γ_w) ($y = d/2$) is,

$$\gamma w = -(\Delta P/(l\,\eta))(d/2) \quad (4.52)$$

The both sides of the Eq. (4.51) are integrated by y.

$$dv = -(\Delta P/(l\,\eta))y\,dy \quad (4.53)$$
$$\int dv = -(\Delta P/(l\,\eta))\int y\,dy \quad (4.54)$$
$$v = -(\Delta P/(2\,l\,\eta))y^2 + D \quad (4.55)$$

Eq. (4.55) represents the flow velocity distribution in the cross section. D is a integration constant. The flow velocity is zero at the wall ($v = 0$ at $y = d/2$).

$$D = (\Delta P/(8\,l\,\eta))d^2 \tag{4.56}$$

Eq. (4.56) is substituted into Eq. (4.55),

$$v = (\Delta P/(8\,l\,\eta))(d^2 - 4y^2) \tag{4.57}$$

Eq. (4.57) represents a parabolic flow velocity distribution. By integrating the flow velocity in the channel cross-section, the flow rate is calculated.

$$Q = \int_0^{\frac{d}{2}} 2b\,\frac{\Delta P}{8l\eta}(d^2 - 4y^2)dy \tag{4.58}$$

$$Q = \frac{b\,\Delta P}{4l\eta}\left[d^2 y - 4\frac{y^3}{3}\right]_0^{\frac{d}{2}} \tag{4.59}$$

$$Q = b\,d^3\Delta P/(12\,l\,\eta) \tag{4.60}$$

By modifying Eq. (4.60),

$$\Delta P = 12\,l\,\eta\,Q/(b\,d^3) \tag{4.61}$$

Eq. (4.61) is substituted into Eq. (4.52),

$$\gamma w = 6\,Q\,/(b\,d^2) \tag{4.62}$$

Under the flow between parallel walls, the adhesive strength between the cells and the wall of the scaffold can be estimated by the wall shear stress at the separation of the cells from the plates (**Fig. 4.32**) [14]. Microscopic observation of cells in the channel between parallel walls (**Figs. 4.33, 4.34**) enables quantitative evaluation of the behavior of cells: the effect of shear stress on migration, deformation, proliferation, orientation and differentiation of cells (**Fig. 4.35**, **Fig. 4.36**) [27].

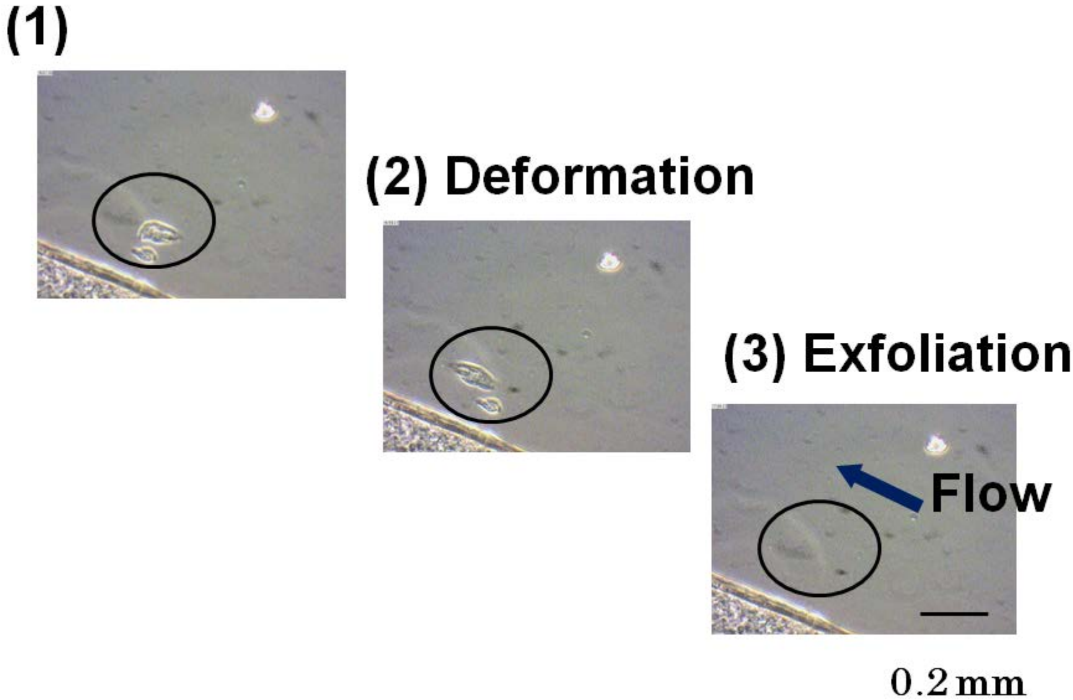

Fig. 4.32: Deformation and exfoliation of cell in flow.

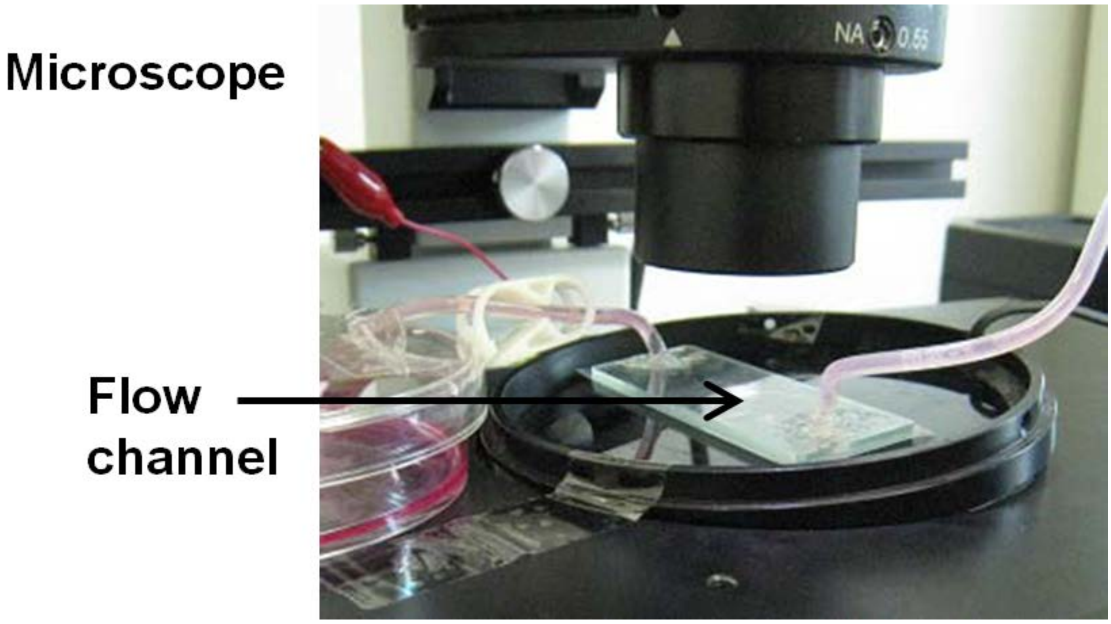

Fig. 4.33: Flow channel between parallel walls.

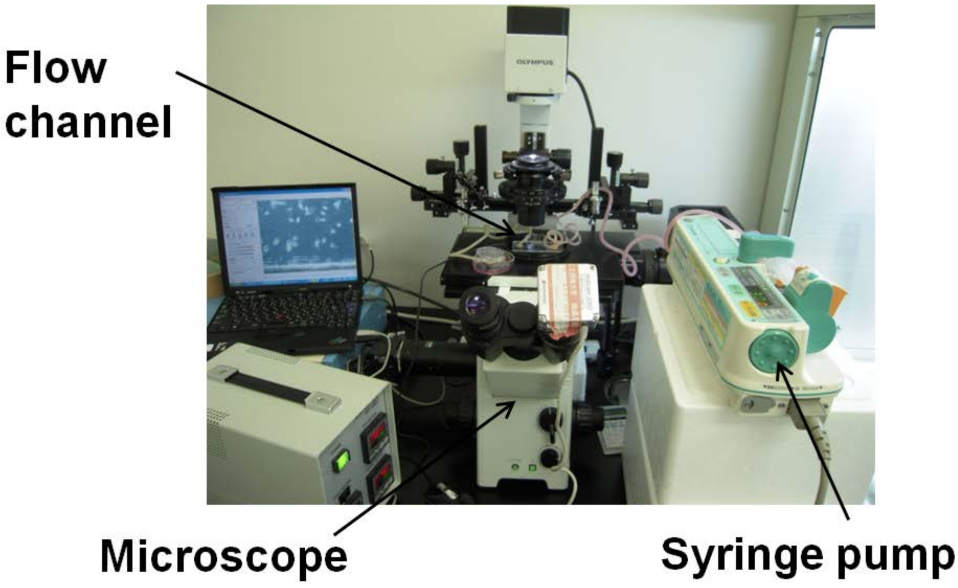

Fig. 4.34: Flow channel system with parallel wall for microscopic observation.

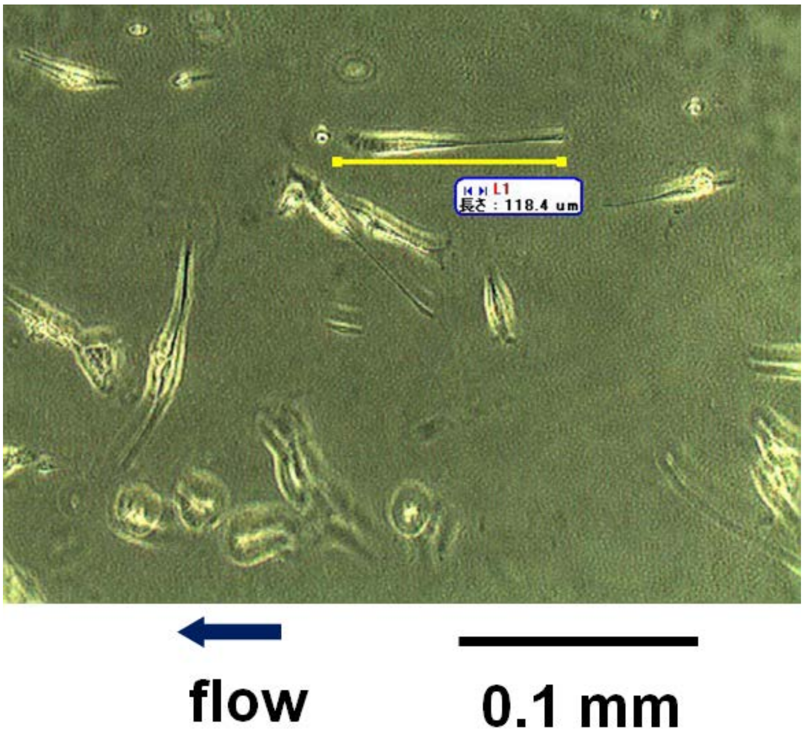

Fig. 4.35: Extension of cell.

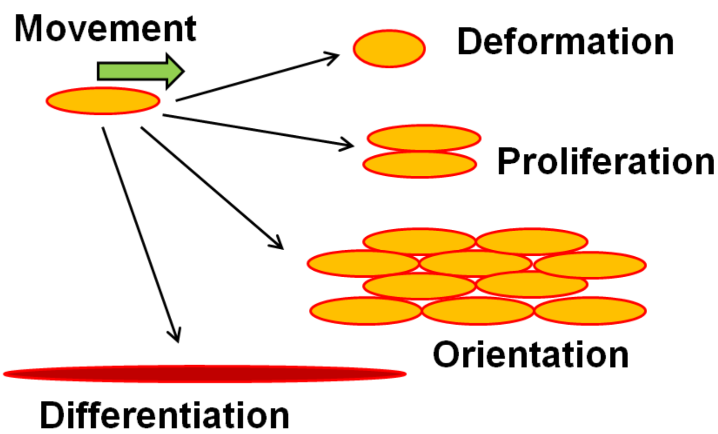

Fig. 4.36: Movement, deformation, proliferation, orientation, and differentiation of cell.

4.2.6 Secondary flow

The viscosity of a fluid around a ball can be measured by the velocity of the ball falling by the gravity (**Fig. 4.37**).

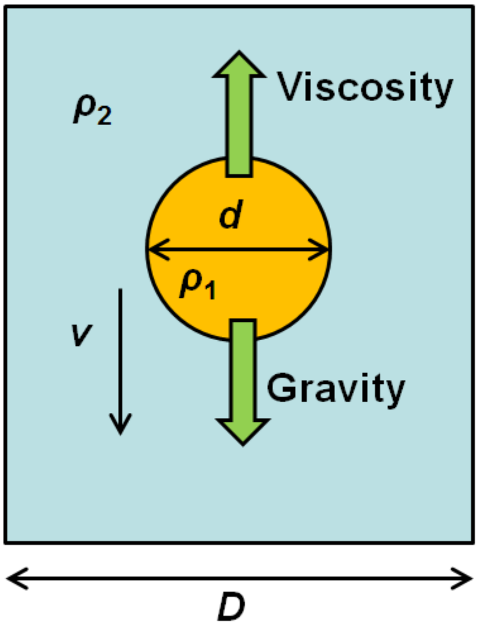

Fig. 4.37: Falling sphere.

$$\eta \propto d^2(\rho_1 - \rho_2) / v \tag{4.63}$$

In Eq. (4.63), η is viscosity, d is diameter of the ball, v is falling velocity of the ball, and $\rho_1 - \rho_2$ is the difference between the density of the ball and the fluid.

The influence of the tube wall is related to the diameter ratio d / D between the ball and the circular tube. Eq. (4.63) is established, when turbulence (see 4.3.2) does not occur around the sphere.

Consider the flow around the moving sphere with rotation. Fluid in the vicinity of the surface of the rotating sphere is dragged to the rotational direction. In the region where the direction of dragging is same as that of moving, the density of the stream line increases (see 4.3.2). In the region of the higher density of the stream line, the pressure reduces (see 4.1.1, see Fig. 4.4). By the reduction of the pressure, the ball receives the force vertical to the moving direction (Fig. **4.38**).

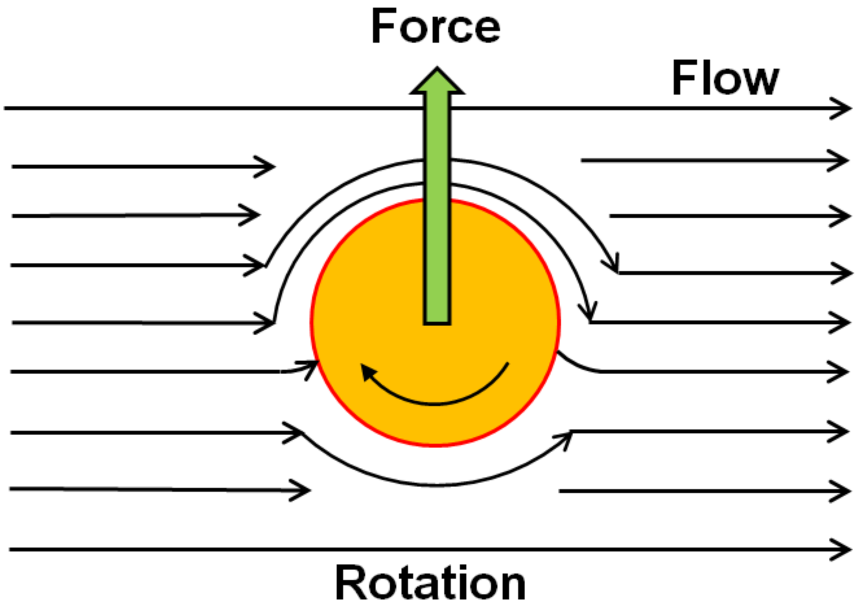

Fig. 4.38: Magnus effect.

The cylinder rotating in the flow receives the similar effect. The relative movement of the rotating ball in the flow is same as the rotating ball moving in the fluid. The effect is called “**Magnus effect**”. The effect explains the curving direction of the movement of the rotating ball.

The suspended particles rotate and shift to the center axis in the velocity distribution in the pipe, where the velocity is high at the center and low near the wall. This is called “**concentration to axis**”. Behavior of suspended particles depends on the density of particle relative to that of the surrounding fluid. By the effect, erythrocytes flow near the central axis, and they do not flow in the vicinity of the wall (**Fig. 4.39**). In this case, the destruction of erythrocytes decreases in the low shear field near the center.

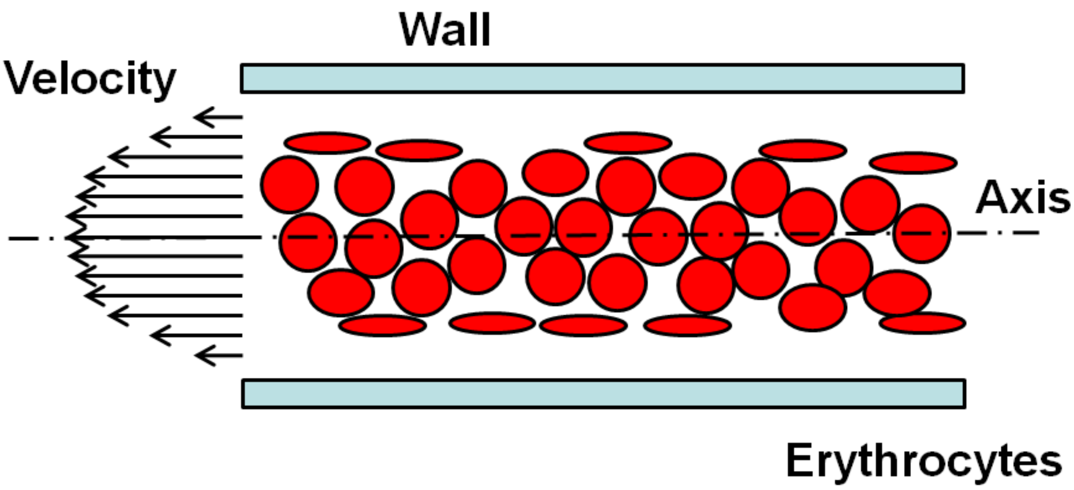

Fig. 4.39: Axis concentration.

In the bending pipe, the vertical component at the main flow direction is generated by the inertia (**Fig. 4.40**). The flow component is called “**the secondary flow**”.

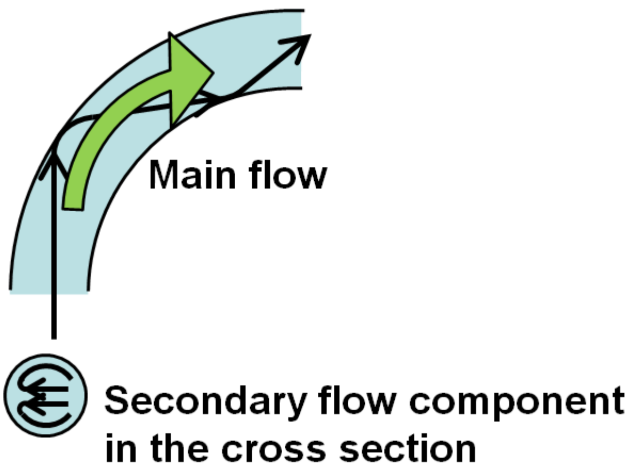

Fig. 4.40: Secondary flow in bend tube.

Consider the flow in the fluid sheared between “the side surface of the rotating inner cylinder” and “the side surface of the stationary outer cylinder”, which have the common axis of symmetry. The Couette type flow generates between the inner cylinder and the outer cylinder. The fluid at the vicinity of the inner cylinder is directed to the outer cylinder by the centrifugal force. The flow to the outside from the inside generates the continuous spiral secondary flow (**Fig. 4.41**). The spiral flow is called "**Taylor vortex**". Also in the flow between the rotating cone and the stationary plate (see 4.2.4), the secondary flow of Taylor type vortex is generated by the centrifugal force (**Fig. 4.42**).

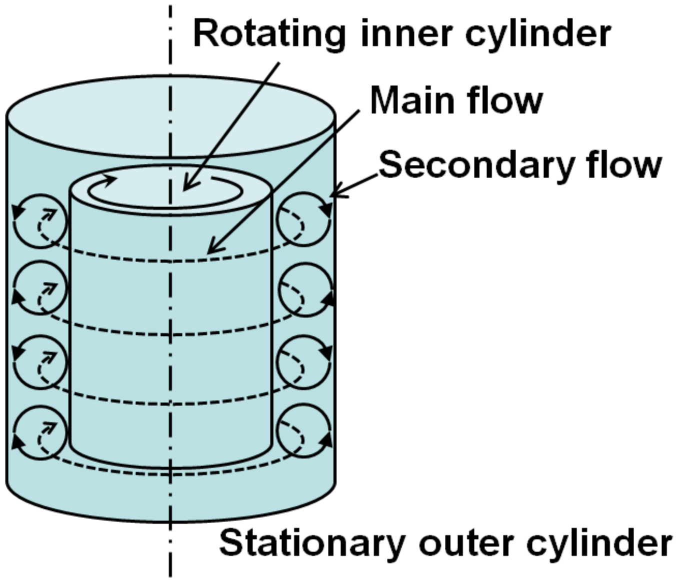

Fig. 4.41: Secondary flow between cylinder (Taylor vortex).

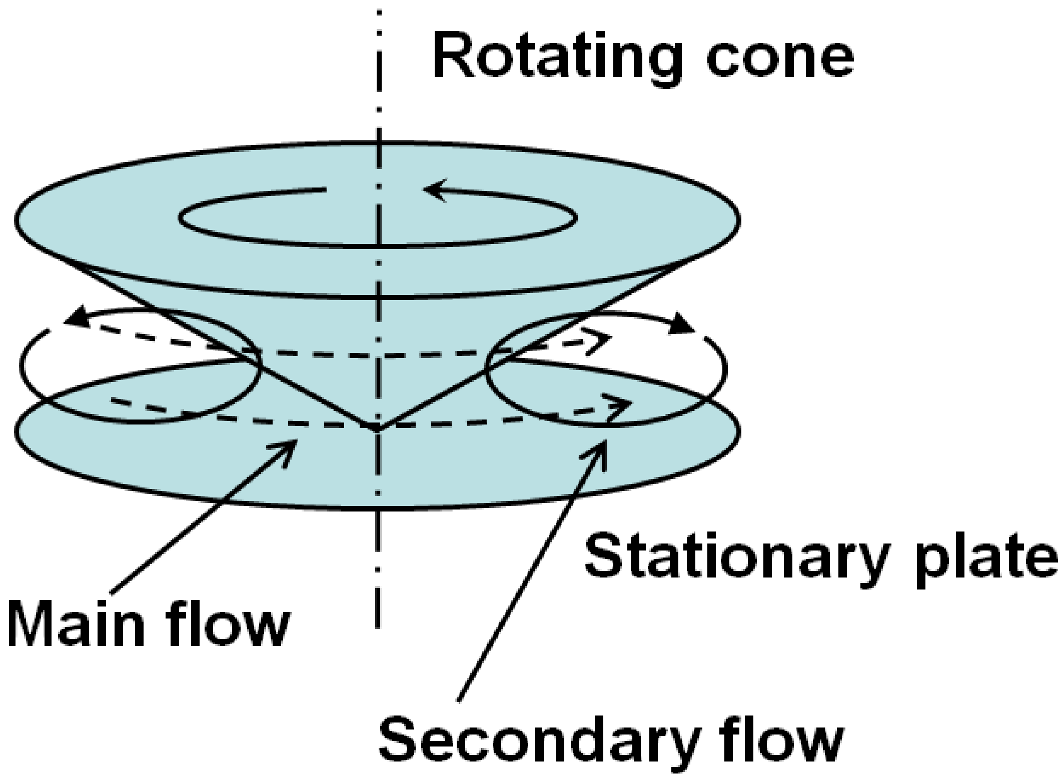

Fig. 4.42: Secondary flow between rotating cone and stationary plate.

The secondary flow with the centrifugal action can be decreased, on the other hand, by the outside rotation rather than the inside rotation: sandwich the fluid between the stationary inner cylinder and the rotating outside cylinder (**Fig. 4.43**), or between the rotating concave cone and the stationary convex cone (**Fig. 4.44**).

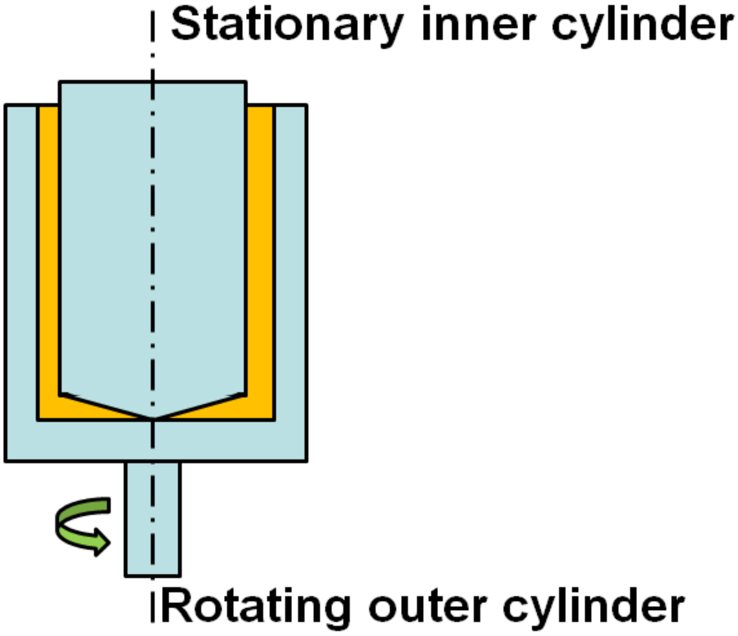

Fig. 4.43: Flow between rotating outer cylinder and stationary inner cylinder.

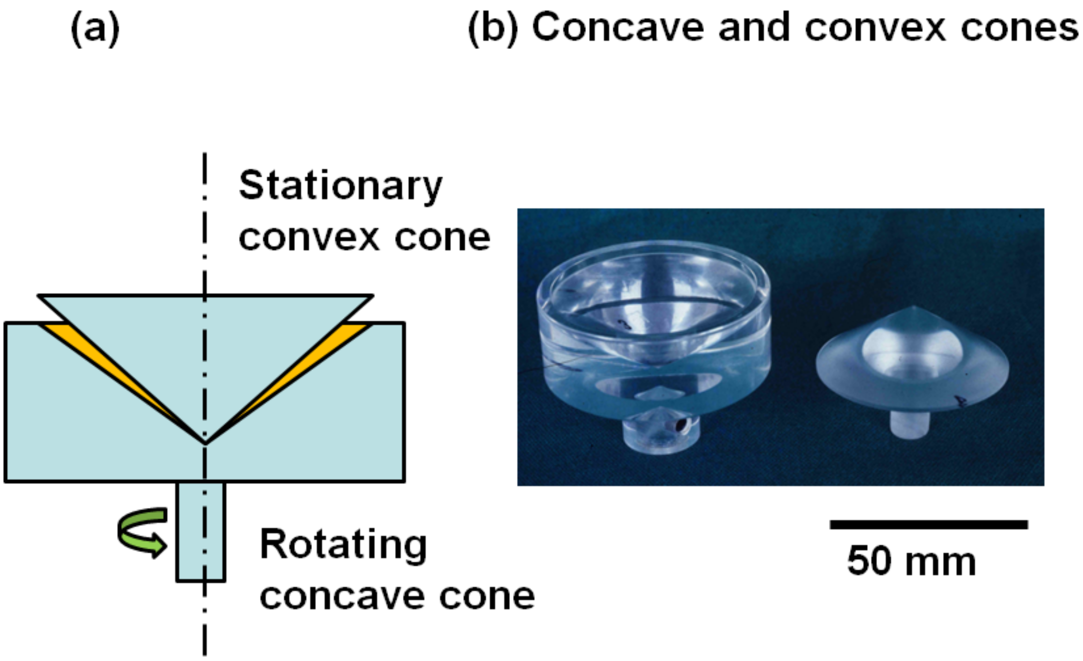

Fig. 4.44: Flow between stationary convex cone and rotating concave cone.

The convex conical bottom surface of the stationary inner cylinder induces the uniform shear field even at the bottom (Fig. 4.43). The modification decreases the secondary flow. The uniform shear rate throughout the fluid is effective for fatigue failure testing of erythrocytes [15].

4.3 Steady flow and non-steady flow

4.3.1 Pulsatile flow

A flow, of which the velocity does not change with time, is called "**steady flow**". A flow, of which the velocity changes with time, is called "**non-steady flow**". The velocity of blood in the artery changes periodically correspond with the beat of the heart. This kind of flow is called "**pulsatile flow**", or "**pulsating flow**". Both pulsatile flow and pulsating flow are included in the non-steady flow.

Do you remember the equation of motion? The flow should be accelerated, when the forces applied on the fluid are not balanced (see 4.2.2). In a steady flow, the force of the pressure difference is balanced with that of resistance (Fig. 4.21). When the flow resistance is zero, the fluid flow continues without a pressure difference.

In the pulsatile flow, on the other hand, acceleration and deceleration are alternatively repeated. During acceleration, the force due to the pressure difference exceeds the force of resistance. During deceleration, the force resistance exceeds the force due to the pressure difference.

For example, consider the flow resistance at the aortic valve. In the first half of the systole (accelerated phase), the left ventricular pressure exceeds the aortic pressure. The force due to the pressure difference is greater than that of the resistance in the aortic valve. The flow from the left ventricle into the aorta is accelerated (**Fig. 4.45**).

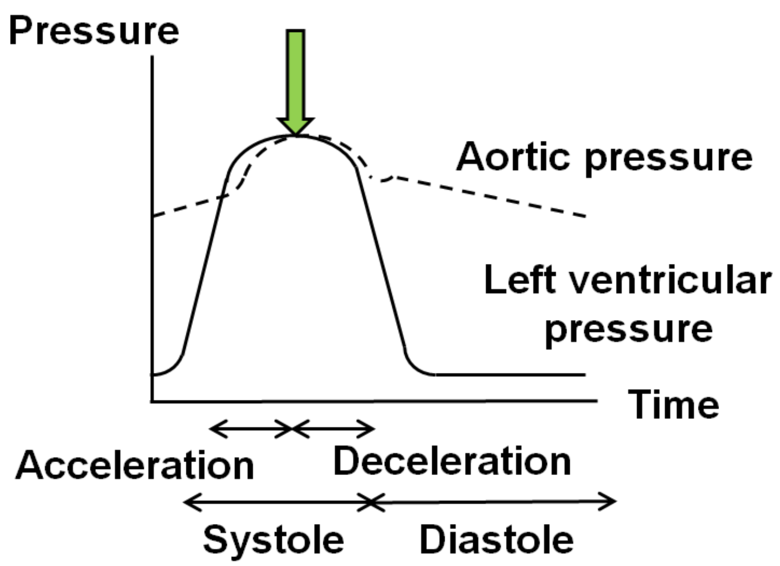

Fig. 4.45: Pressure in pulsatile flow.

Subsequently, in the second half of the systole (deceleration phase), the left ventricular pressure often falls below the aortic pressure (after the arrow in Fig. 4.40). In other words, the pressure is higher at downstream than upstream. The pressure difference decreases flow velocity. The aortic valve closes, before the back motion of the flow. The resistance of the aortic valve is very small, because of the expansion of annulus of the aortic valve during opening (systolic phase of the heart).

When the flow path resistance at the aortic valve is large, the left ventricular pressure exceeds the aortic pressure even in the deceleration phase. The large flow path resistance decelerates the flow. The greater flow resistance at the aortic valve can be estimated by the timing, when the left ventricular pressure exceeds the aortic pressure in the deceleration period. It is applied to the estimation of aortic valve stenosis.

The intravascular pressure fluctuates in the pulsatile flow. The fluctuation of the internal pressure makes variation of the internal diameter of the vessel according to the compliance of the vessel wall. The local compliance governs the local wall deformation, which might make a step at the wall in the flow path (**Fig. 4.46**). The step at the connection of the artificial blood vessel with the native blood vessel causes thrombus formation. To avoid clot formation, the compliance of the wall should be adjusted to that of the native.

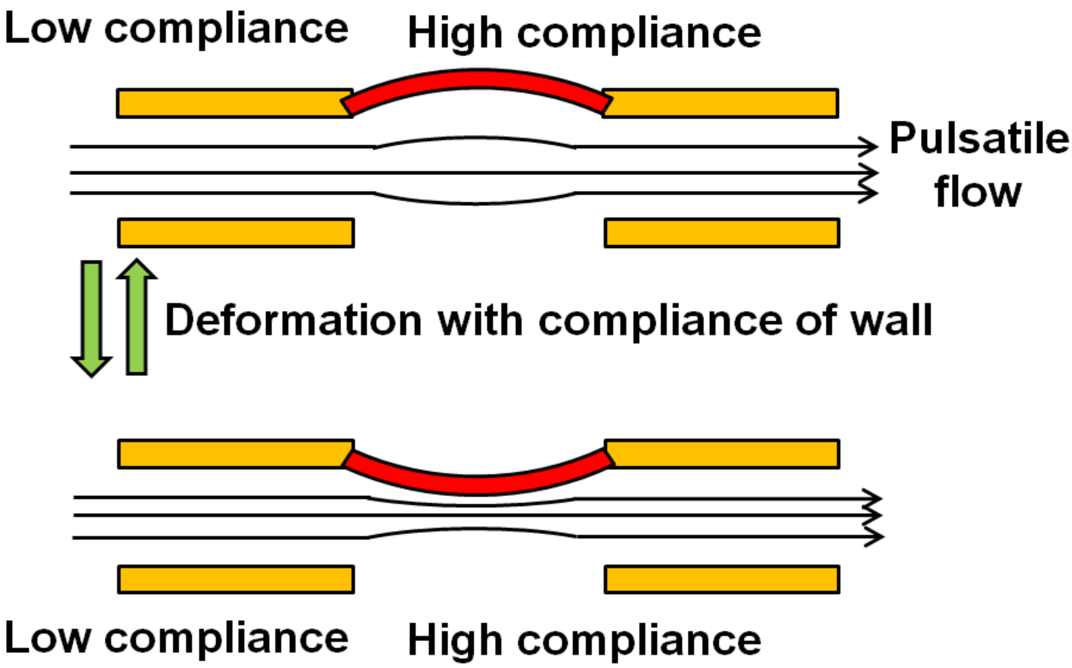

Fig. 4.46: Compliance of tube wall.

In the pulsatile flow, the periodical low shear rate inhibits the destruction of red blood cells [15]. Since the stirring effect of the pulsatile flow washes away the stagnation region, the clot growth can be inhibited [28]. The periodical high wall shear rate inhibits thrombus growth [29] (**Fig. 4.47**). Effect of the flow on the adhesion of endothelial cells to the vascular wall is different between in the pulsatile flow and in the steady flow [30].

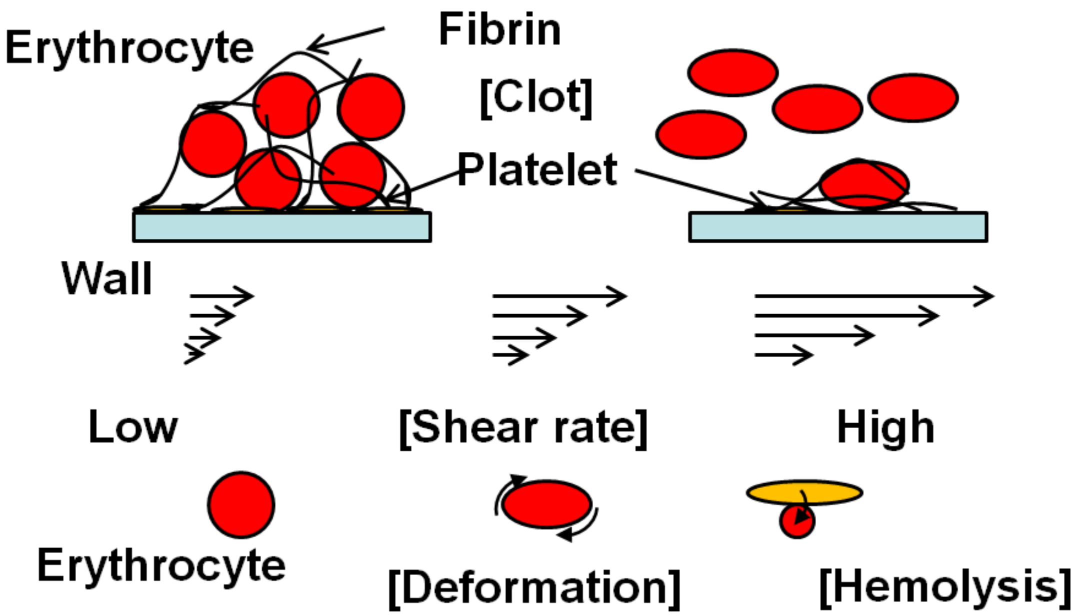

Fig. 4.47: Clot formation and hemolysis with shear rate.

4.3.2 Laminar flow and turbulent flow

When the inertial force is dominant than the viscous force, each layer in flow moves independently and mixed with each other. The flow with no mixture between layers is called "**laminar flow**". The flow with mixture between layers, on the other hand, is called "**turbulent flow**" (**Fig. 4.48**).

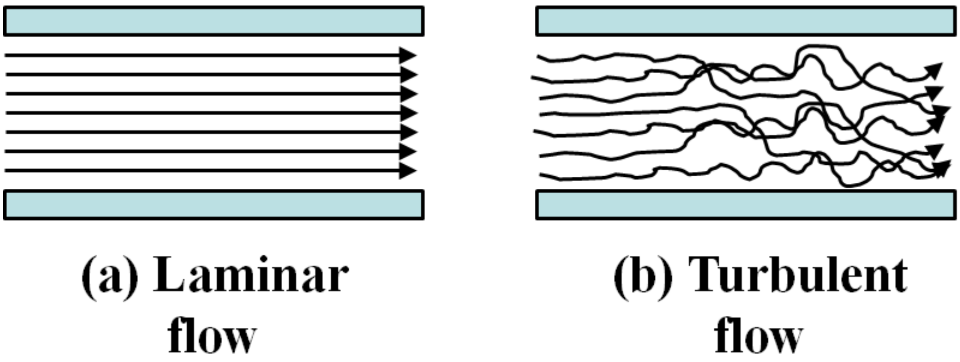

Fig. 4.48: Tracing.

The ratio of “**inertial force**” per “**viscous force**” is called “**Reynolds number** (*Re*)”.

$$Re = \rho v\, x / \eta \qquad (4.64)$$

In the equation (4.64), ρ is the density, v is the representative velocity, x is the representative length, and η is the viscosity coefficient.

Reynolds number is dimensionless (see 2.1.1), and has no units (see Q. 4.8), since it is a ratio of the "force" to "force". Reynolds number is an indicator for predicting whether the flow is laminar or turbulent. The Reynolds number, at which the flow transitions from laminar to turbulent, is called as the “**Critical Reynolds number**”.

As the dimensionless number, Reynolds number is applied to the result of the experiment at miniature model to the prediction of the flow at the actual size product (**similarity law)**. The simulation is planning at the same the Reynolds number. The turbulent flow occurs at the higher Reynolds number: the larger diameter of the blood vessels, the higher velocity of the flow, and the lower viscosity. With the average flow velocity and the inside diameter at the various blood vessels, the Reynolds number in the human blood circulation system is calculated between 0.0005 and 2000 [24].

At the steady flow in the sufficiently long straight pipe, v of the average flow velocity at the cross-section, and the x of the inner diameter of the pipe, the critical Reynolds number is in the range between 2000 and 4000. Since the blood flow in the artery is "pulsatile", the blood flow in the artery tends to generate turbulence.

A flow is able to be described by two ways of methods: tracing the velocity at a fixed position, or tracing the velocity of a flowing particle. The state of flow can be visualized by the movement of the dye or the micro particles.

The line connecting the direction of flow is called as a **streamline** (**Fig. 4.49**). In the laminar flow of the steady flow, streamline is consistent with the flow of the trajectory. Streamlines do not intersect. When they intersect, the direction of flow cannot be defined at the intersection.

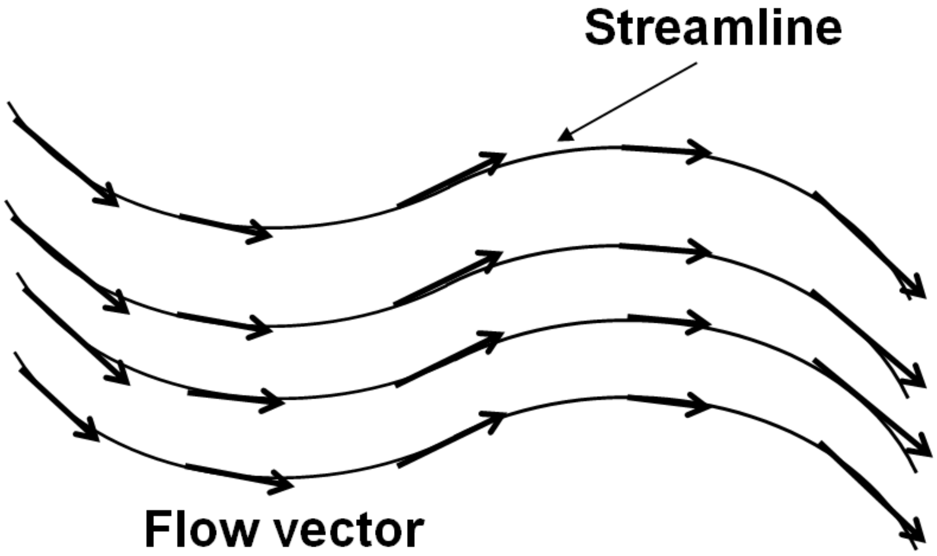

Fig. 4.49: Streamline.

Both in the turbulent and in the unsteady flow, the streamline varies with the variation of the flow velocity vector. In these flows, the streamline at each moment does not coincide with the tracings of the flow.

When the Reynolds number increases, the flow at each position in the fluid becomes unsteady. This is because non-steady peeling-vortex is generated by the inertia of the fluid (**Fig. 4.50**). In the downstream of the obstruction, the peeling vortex is formed alternately, which is called **Karman vortex**.

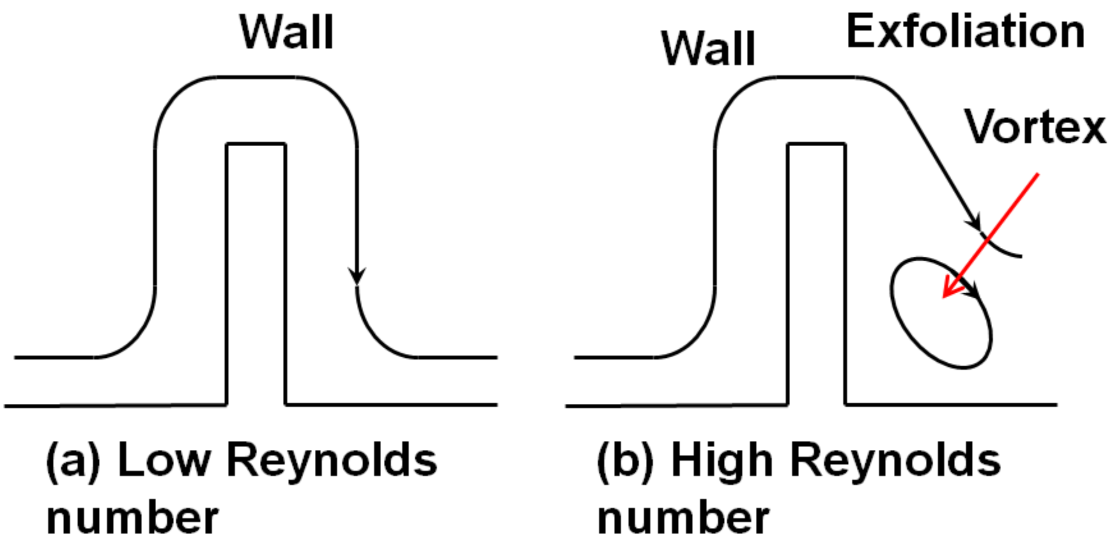

Fig. 4.50: Vortex.

A smooth wall of the flow path prevents the flow separation. By smoothing the inner surface of the artificial ventricles, the disturbance of the blood flow can be prevented (**Fig. 4.51**). The directions of the inflow and the outflow of the artificial ventricle change the flow pattern in the artificial ventricle. The flow pattern has an effect on the thrombus formation (**Fig. 4.52**) [31].

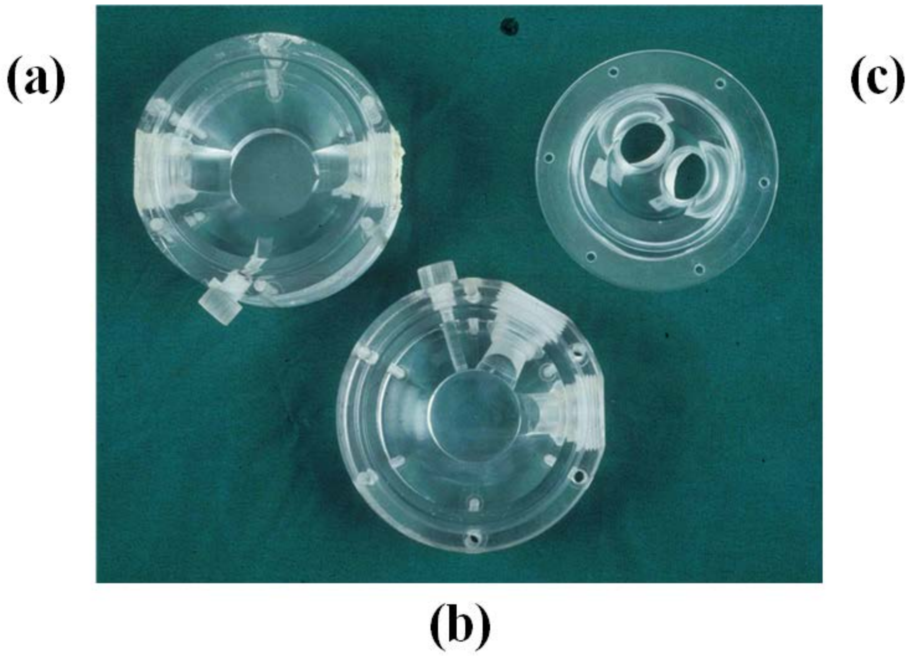

Fig. 4.51: Artificial ventricle.

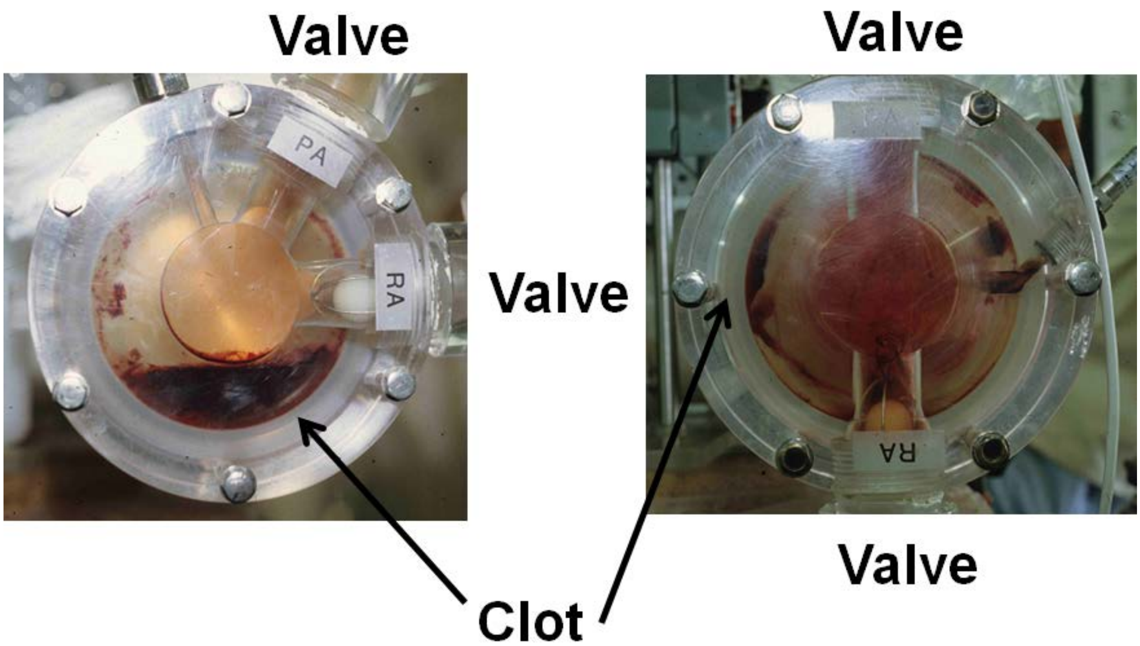

Fig. 4.52: Clot in artificial ventricle.

The mixing movement between layers increases the flow resistance in the turbulent flow. The stirring effect in the turbulent flow, on the other hand, increases the transport efficiency of the material. The agitating action of the fluid in the vicinity of the membrane increases the efficiency of the gas exchange in the oxygenator (the artificial lung) (see 5.3.1).

Questions

Q 4.1: Why you cannot apply the simple Kelvin-Voigt model to analysis of stress tracings, when you deform material by a stepwise strain?

Q 4.2: The "Newtonian Fluid" is flowing through a sufficiently long straight cylindrical pipe. How much decrease the resistance of the laminar flow, when the inner diameter doubles?

Q 4.3: List cases in which the vascular resistance increases.

Q 4.4: Calculate the resistance of the laminar flow of the water through the capillary. The capillary has the Inner radius $a = 1 \times 10^{-4}$ m and the length $l = 0.1$ m. Let the coefficient of viscosity of the water $\eta = 1 \times 10^{-3}$ Pa s.

Q 4.5: Calculate the resistance of the pulmonary circulation, when the mean pulmonary arterial pressure is 25 mmHg, the left atrial pressure is 5 mmHg, and the cardiac output is 5.5 l min^{-1}.

Q 4.6: Why the term of diastole is longer than that of systole in ventricular pumping action?

Q 4.7: In which case does the pulsatile-flow flow against the pressure gradient?

Q 4.8: Show that the Reynolds number is dimensionless from the definition equation.

Q 4.9: Express the torque "T" to rotate the cone at a rotational angular speed "ω", when the fluid of the viscosity coefficient "η" is sandwiched between the cone and the plate, using the radius "R" of the cone and the clearance angle "θ" between the cone and the plate in Figure 4.27. Calculate the torque "T" in the case of the following condition: η = 0.005 Pa s, ω = 6 rad s^{-1}, R = 0.02 m, and θ = 0.02 rad.